WISHEPP
4th Winter School in HEP
November 9-15, 2019 An-Najah N. University, Nablus, Palestine
Bruno Mazoyer LAL 2019

Gravitational waves

→ Lecture 1
Introduction to gravitational waves

→ Lecture 2
Detection of gravitational waves

→ Lecture 3
Multi-messenger astronomy

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Introduction to gravitational waves

- → General relativity
- → Gravitational waves
- → First detections of gravitational waves
- → Characterization of black hole binary systems

Illustration

CNRS LAL Laboratoire de l'Accélérateur Linéaire

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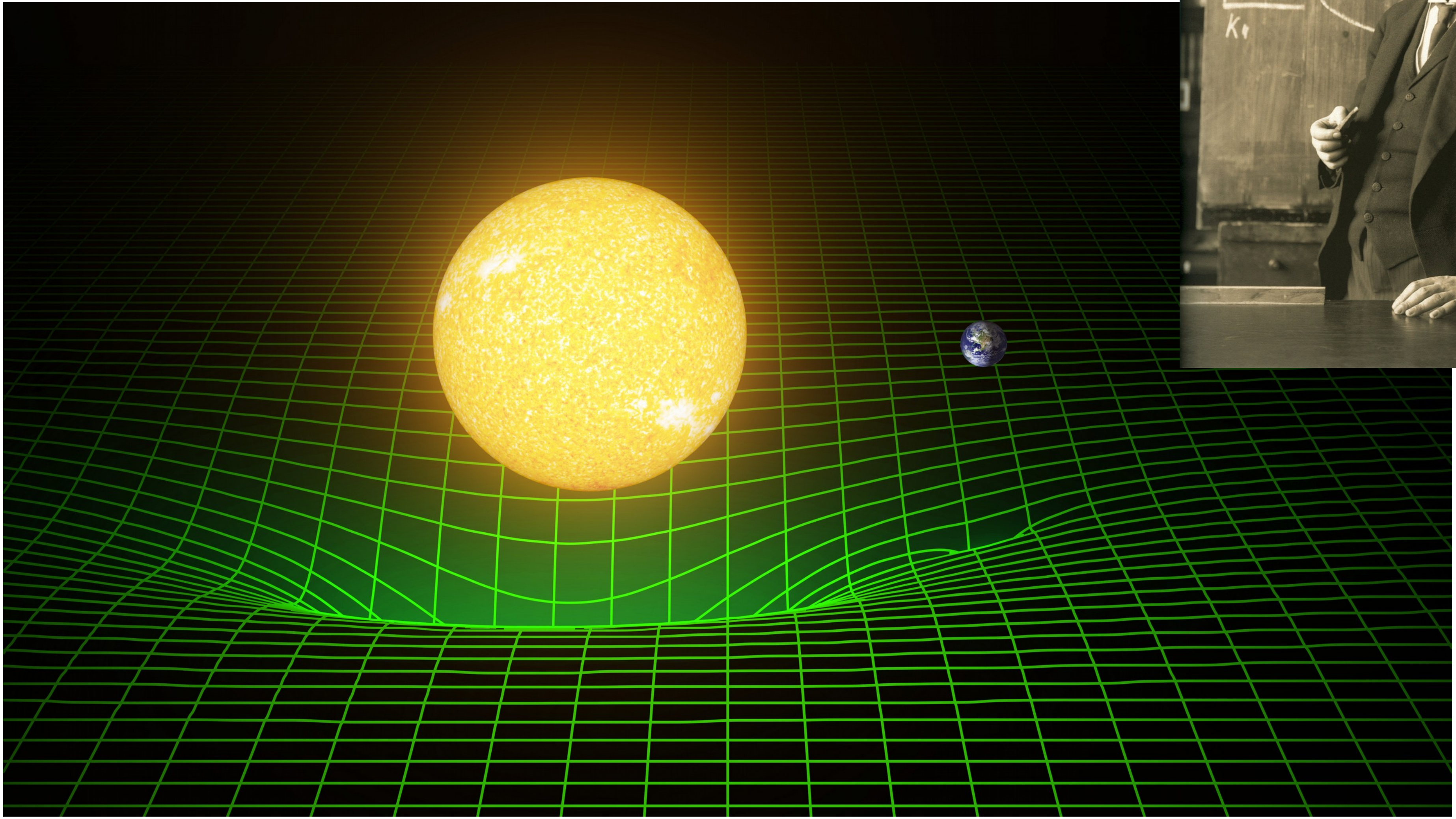
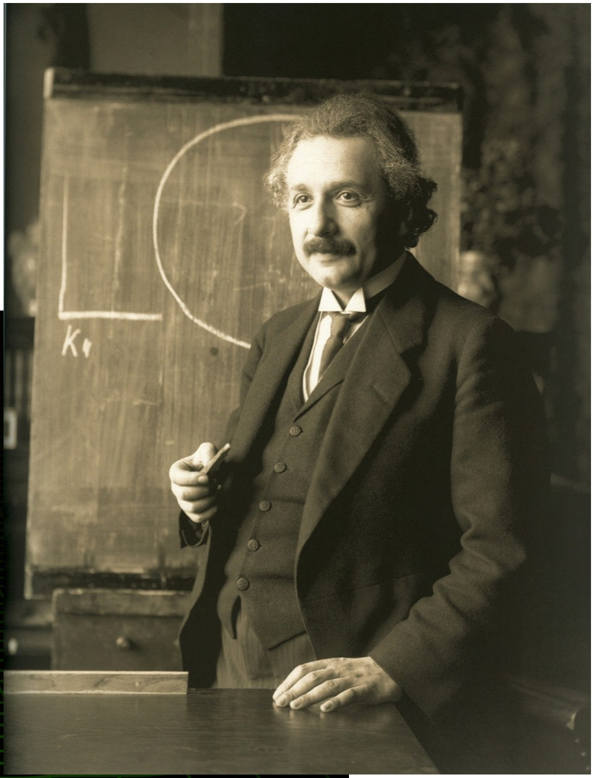
General Relativity

- **1915:** The theory of general relativity is published by Albert Einstein
- Current description of gravitation
- Superior to Newtonian gravity
- Gravity **=** geometric description of space and time





General Relativity

Metric: space-time structure, used to define distances

Space-time is described by the metric tensor $g_{\mu\nu}$

Distances are measured by integrating the distance element:

$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}$

Example: Minkowski flat metric (empty space, c=1)

time

$$g_{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} \boxed{-1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad ds^2 = -dx^0 dx^0 + dx^1 dx^1 + dx^2 dx^2 + dx^3 dx^3$$

Euclidean metric

In presence of gravity, the metric is curved

→ distance = geodesics

General Relativity

Einstein's equation:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Space-time curvature ⟺ Mass/energy

General Relativity

Einstein's equation:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \left(+ \Lambda g_{\mu\nu} \right) = \frac{8\pi G}{c^4} T_{\mu\nu}$$

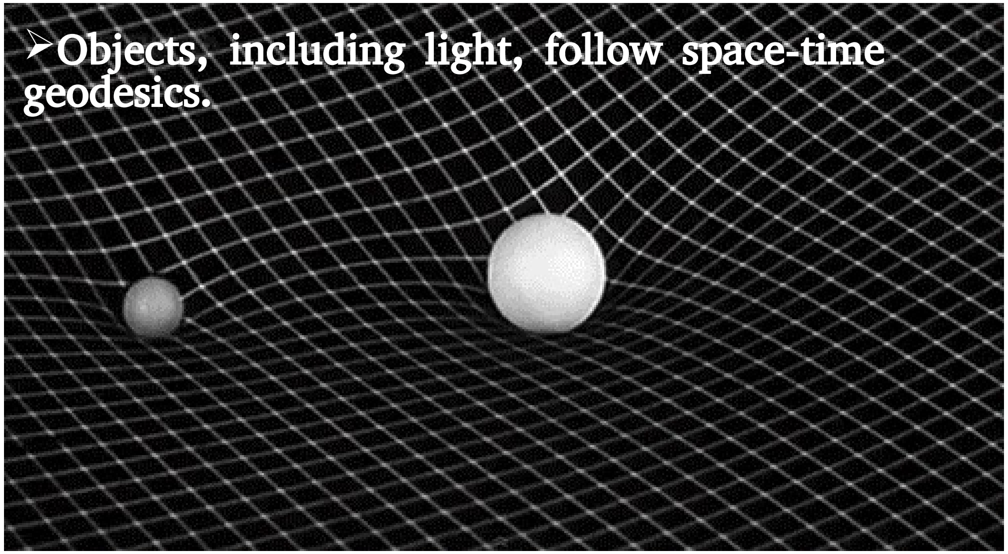
Space-time curvature ⟷ Mass/energy

$R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}$ Ricci tensor = contraction of Riemann tensor

$R = R^{\alpha}{}_{\alpha}$ Scalar curvature: Ricci tensor contraction

$T_{\mu\nu}$ Energy-momentum tensor: density and flux of energy and momentum

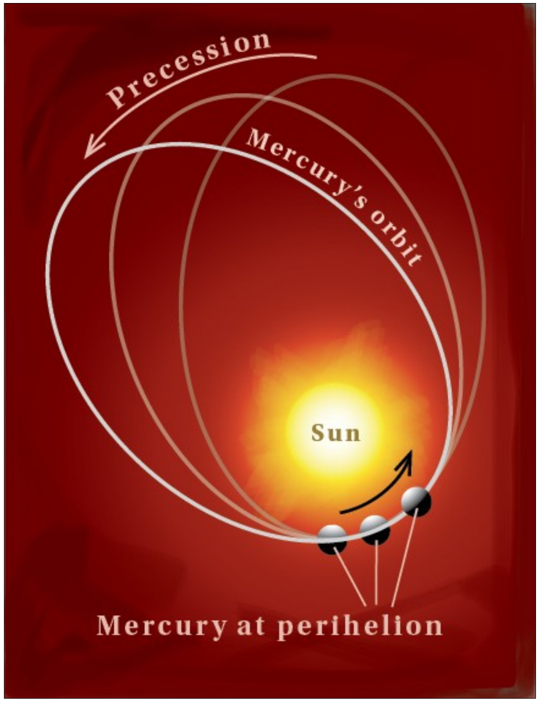
- The entire theory is encoded in a single expression!
- Symmetrical tensors → 10 equations
- Highly non-linear equations

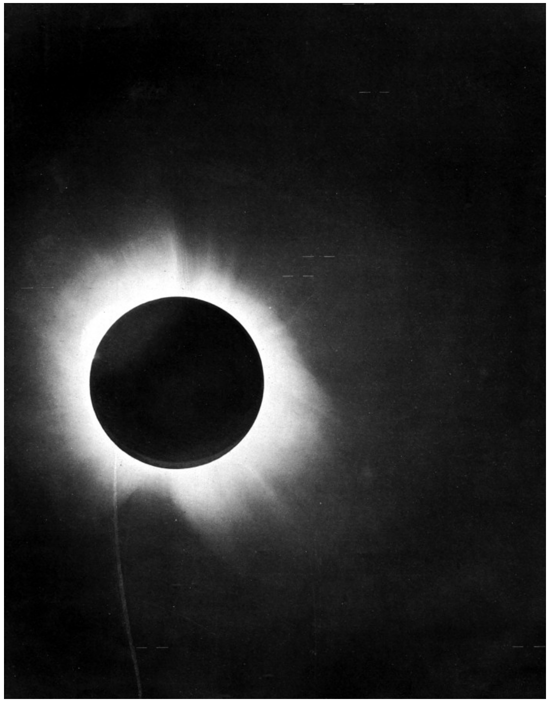


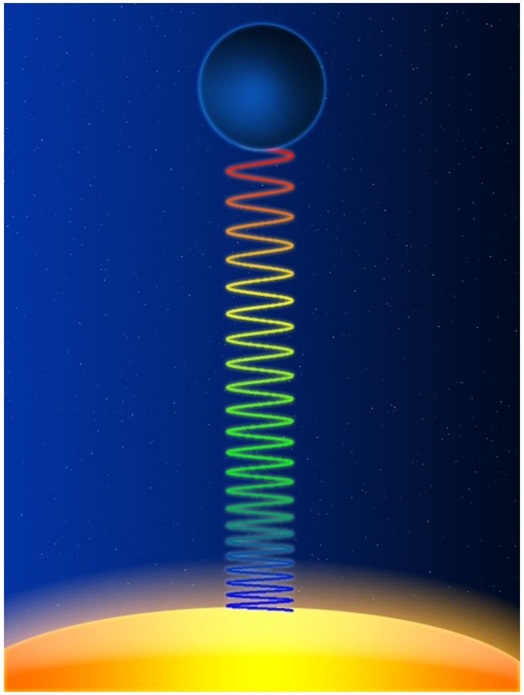
General Relativity

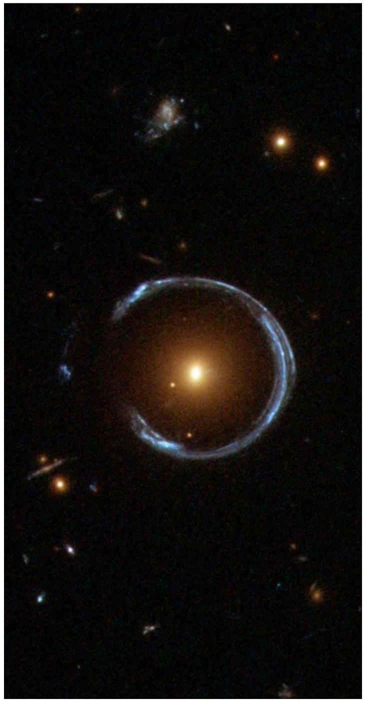
Predictions of the theory

- Anomalous shift (43’’) of the Mercury perihelion
- Light deflection by gravity (observed in 1919)
- Gravitational redshift (observed in 1959)
- Gravitational lensing (observed in 1979)
- Black holes (observed indirectly)
- Gravitational waves (observed in 2015!)









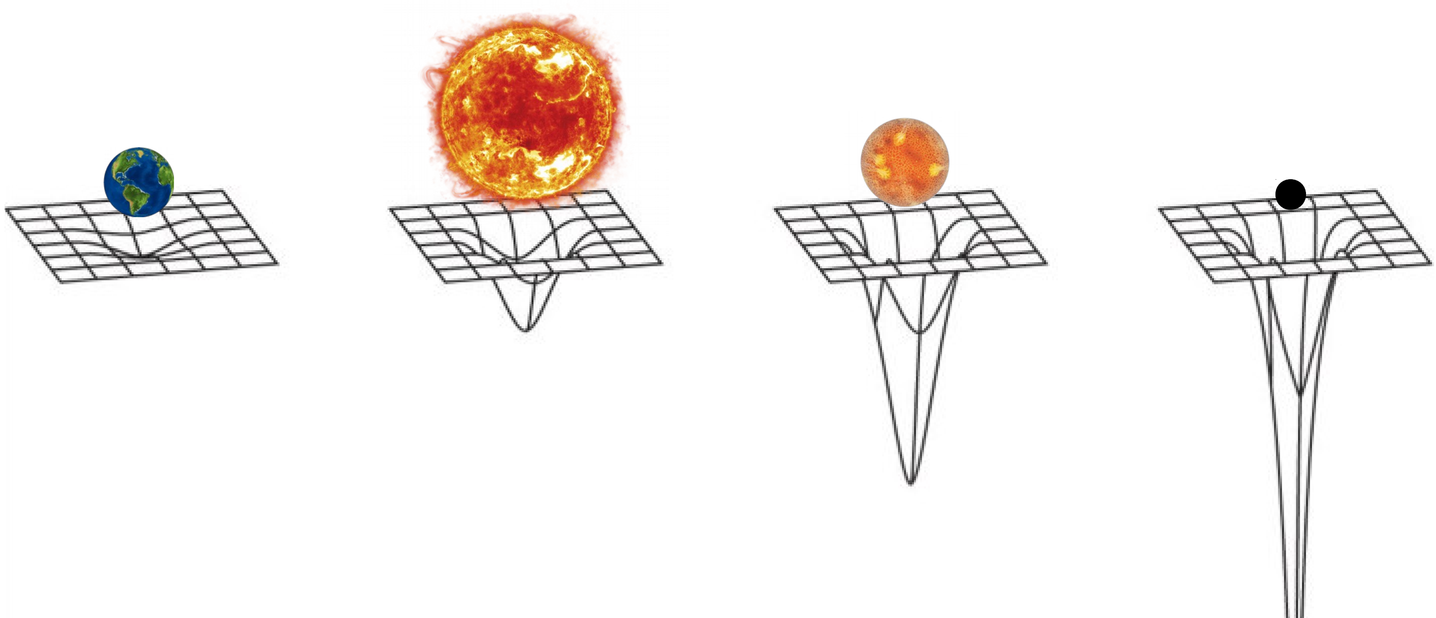
Black holes

Region of space-time deformed by a compact mass from which nothing can escape (not even light). Introduced by Schwarzschild in 1916

Escape velocity (Newton): $v_e = \sqrt{2\frac{Gm}{r}} \xrightarrow{v_e = c} R_s = 2\frac{Gm}{c^2}$ Schwarzschild radius

Black hole: $R < R_s = 2\frac{Gm}{c^2}$

Earth	Sun	Neutron star	Black hole	
$R_s = 9\,mm$	$R_s = 3\,km$	$R_s \sim 5\,km$	$R_s \sim 10\,km$	**Compacity**
$R = 6000\,km$	$R = 700\,000\,km$	$R \sim 10\,km$	$R < R_s$	



Black holes

Theoretical developments in the 60s:

- Rotating black hole solution (Kerr, 1963)
- Electrically charged black hole (Newman, 1965)
- No-hair theorem: mass+spin+charge (1967)
- Singularities as generic solutions (Hawking/Penrose, 1969)

➢ **Stellar black hole** = result from the collapse of a massive star ($m = 3\text{-}100\ M_{sun}$)

➢ **Supermassive black hole** = low-density object at the center of a galaxy ($m \sim 10^9\ M_{sun}$)

➢ **Primordial black hole** = extremely dense object formed just after the big-bang.

Observational evidence:

- star motion near the Milky Way center
- accretion of matter on black holes = bright X-ray sources (X-ray binaries, quasars, AGN)

→ *indirect observations*

Gravitational waves

Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$\boxed{g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu}} \qquad (1)$$

⟸ small perturbation of Minkowski's metric

definierten Größen $\gamma_{\mu\nu}$, welche linearen orthogonalen Transformationen gegenüber Tensorcharakter besitzen, gegen 1 als kleine Größen behandelt werden können, deren Quadrate und Produkte gegen die ersten Potenzen vernachlässigt werden dürfen. Dabei ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$, je nachdem $\mu = \nu$ oder $\mu \neq \nu$.

Wir werden zeigen, daß diese $\gamma_{\mu\nu}$ in analoger Weise berechnet werden können wie die retardierten Potentiale der Elektrodynamik.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} = 0$$

Add a small perturbation to a flat metric:

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $\quad |h_{\mu\nu}| \ll 1$

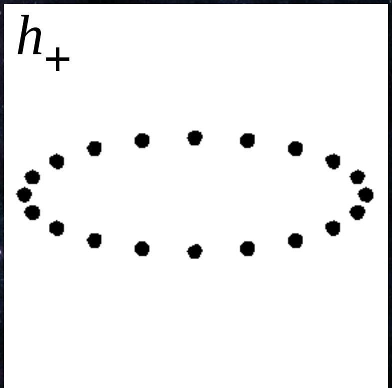
Einstein equations can be linearized and solved:

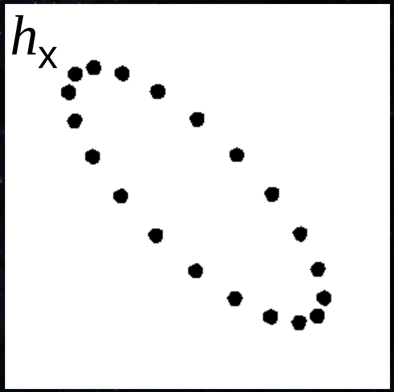
– h obeys a plane-wave equation (transverse-traceless gauge)

– the wave propagates at the speed of light

– 2 degrees of freedom: h_+ and $h_\times$

→ Gravitational waves





Gravitational-wave emission

Monopole	Dipole	Quadrupole (traceless)
$m=\int \rho \, d^3\vec{r}$	$P_i=\int \rho \, x_i \, d^3\vec{r}$	$Q_{ij}=\int \rho \left(x_i \, x_j \right) d^3\vec{r}$

Gravitational-wave emission

~~Monopole~~

$$\cancel{m = \int \rho \, d^3\vec{r}}$$

~~Dipole~~

$$\cancel{P_i = \int \rho \, x_i \, d^3\vec{r}}$$

Quadrupole (traceless)

$$Q_{ij} = \int \rho \left(x_i \, x_j \right) d^3\vec{r}$$

Gravitational-wave emission

~~Monopole~~	~~Dipole~~	Quadrupole (traceless)
$\cancel{m=\int \rho\, d^3\vec{r}}$	$\cancel{P_i=\int \rho\, x_i\, d^3\vec{r}}$	$Q_{ij}=\int \rho\,(x_i x_j)\, d^3\vec{r}$

Einstein quadrupole formula (radiated power)

$$\frac{dE}{dt}=-\frac{G}{5c^5}\left\langle \frac{d^3Q^{ij}}{dt^3}\frac{d^3Q_{ij}}{dt^3}\right\rangle$$

Estimate using the source parameters

$$Q\sim\varepsilon M R^2$$

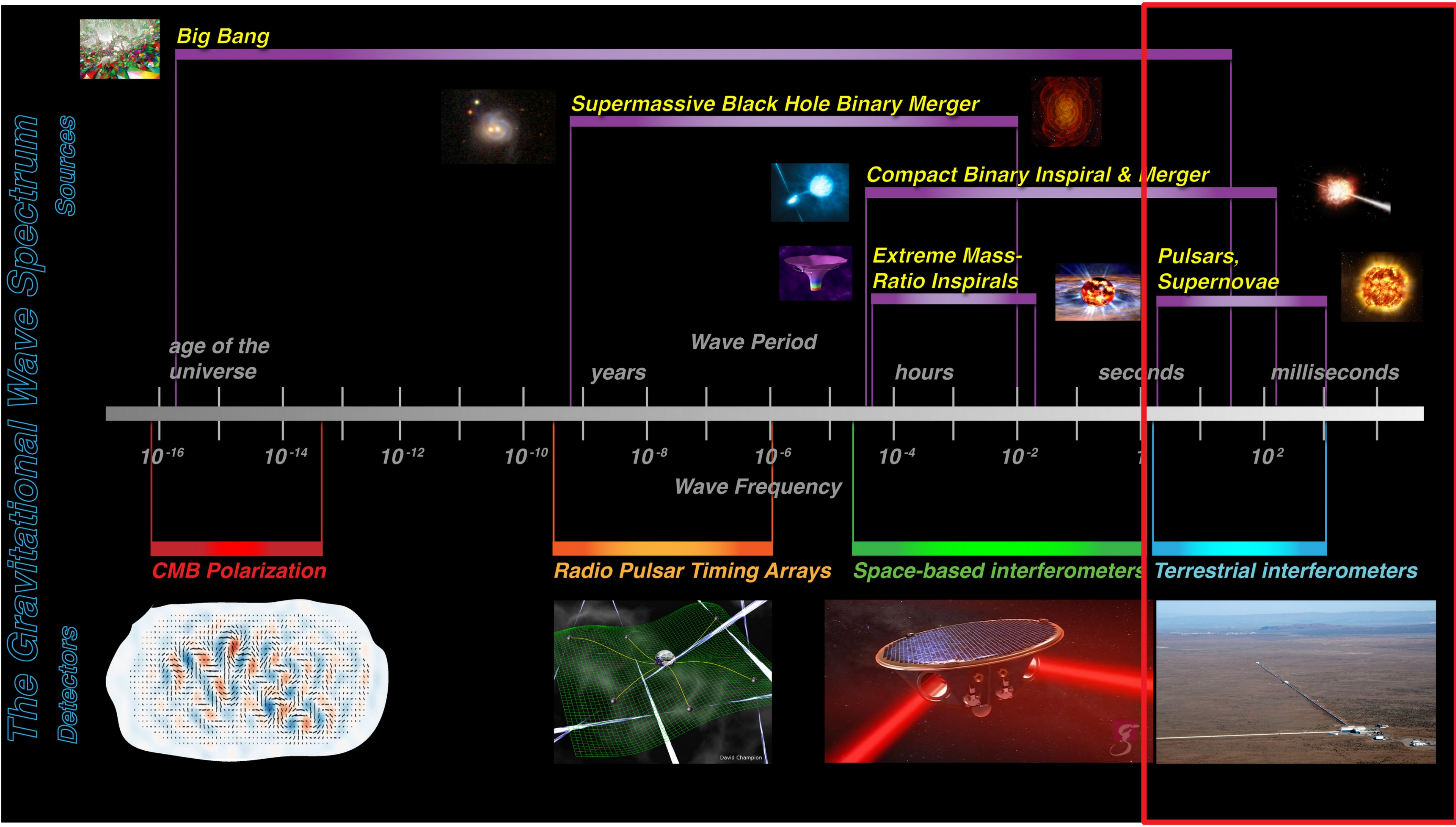
$$\frac{d^3Q}{dt^3}\sim\varepsilon M R^2\omega^3$$

$$\frac{dE}{dt}\sim-\frac{G}{c^5}\varepsilon^2 M^2 R^4\omega^6\sim-\frac{c^5}{G}\varepsilon^2\left(\frac{R_s}{R}\right)^2\left(\frac{v}{c}\right)^6$$

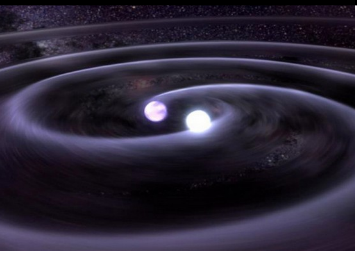
$$\simeq 10^{52}\,W$$

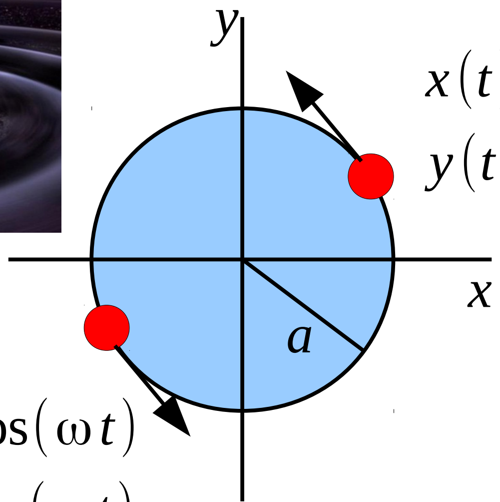
→ Important source characteristics:
- asymmetric
- compact
- relativistic

Gravitational-wave sources



Gravitational-wave emission



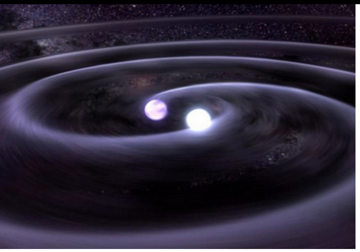


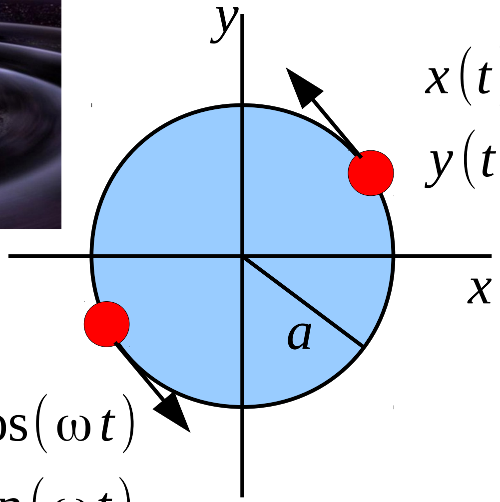
$$x(t) = a\cos(\omega t)$$
$$y(t) = a\sin(\omega t)$$

$$x'(t) = -a\cos(\omega t)$$
$$y'(t) = -a\sin(\omega t)$$

$$Q_{ij} = \int \rho(x_i x_j)\, d^3\vec{r}$$

Gravitational-wave emission





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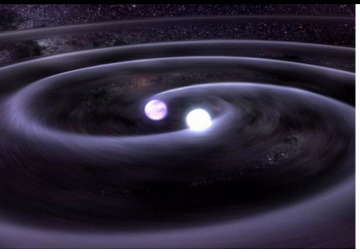
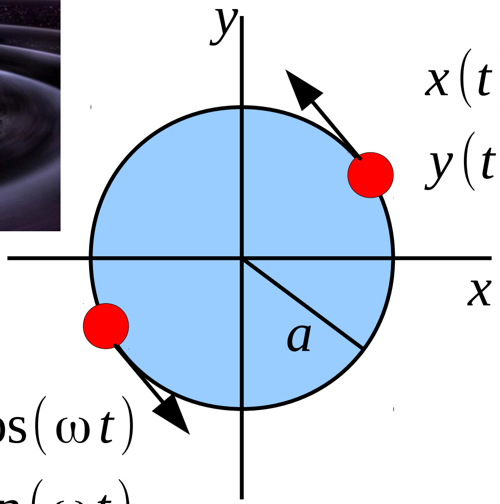
Quadrupolar moment:

$$Q = \begin{pmatrix} ma^2(1+\cos(2\omega t)) & ma^2\sin(2\omega t) \\ ma^2\sin(2\omega t) & ma^2(1-\cos(2\omega t)) \end{pmatrix}$$

Gravitational-wave emission





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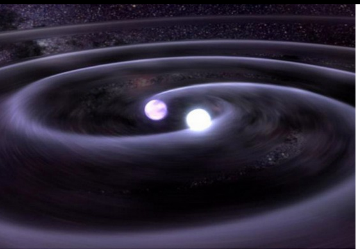
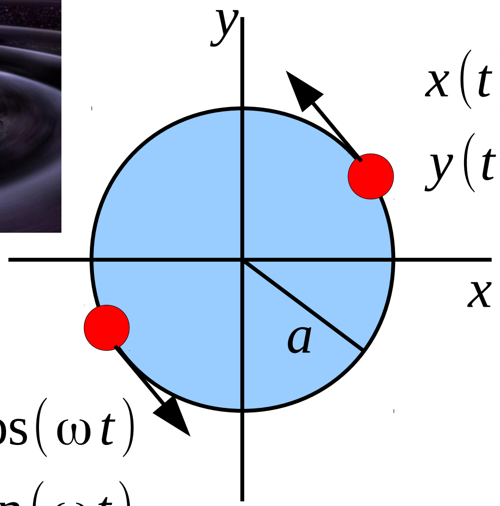
z projection in transverse-traceless gauge:

$$\ddot{Q}=\begin{pmatrix} -4ma^2\omega^2\cos(2\omega t) & -4ma^2\omega^2\sin(2\omega t) \\ -4ma^2\omega^2\sin(2\omega t) & 4ma^2\cos(2\omega t) \end{pmatrix}$$

$$\rightarrow h_{ij}^{TT}=2\frac{G}{rc^4}\ddot{Q}_{ij}$$

Gravitational-wave emission





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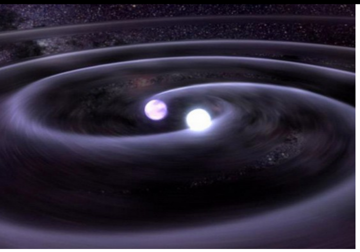
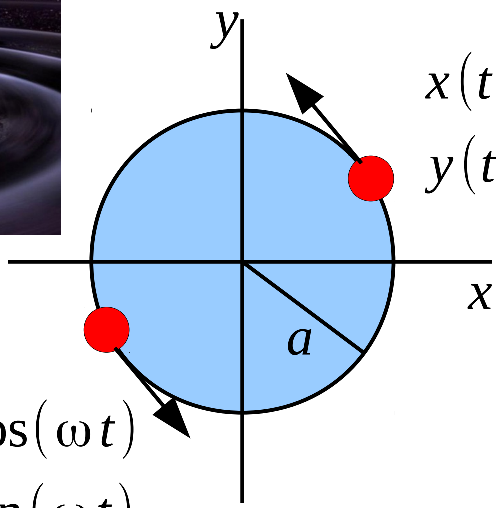
$$\rightarrow h_{ij}^{TT}=2\frac{G}{rc^4}\ddot{Q}_{ij}$$

$$\rightarrow h_+=-2\frac{G}{r\,c^4}4\,\omega^2 ma^2\cos(2\omega t)$$

Gravitational-wave emission





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$$\rightarrow h_{ij}^{TT} = 2\frac{G}{rc^4}\ddot{Q}_{ij}$$

$$\rightarrow h_+ = -2\frac{G}{rc^4} 4\omega^2 ma^2 \cos(2\omega t)$$

1 parsec = 3.26 light-years
100 Mpc = 3.26 x 10^8 light-years

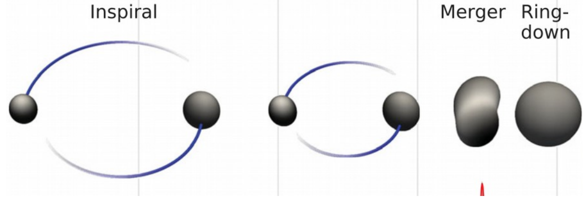
Source at 100 Mpc, rotating at 50 Hz, m=2 M_{sun} orbiting at 1000 km:

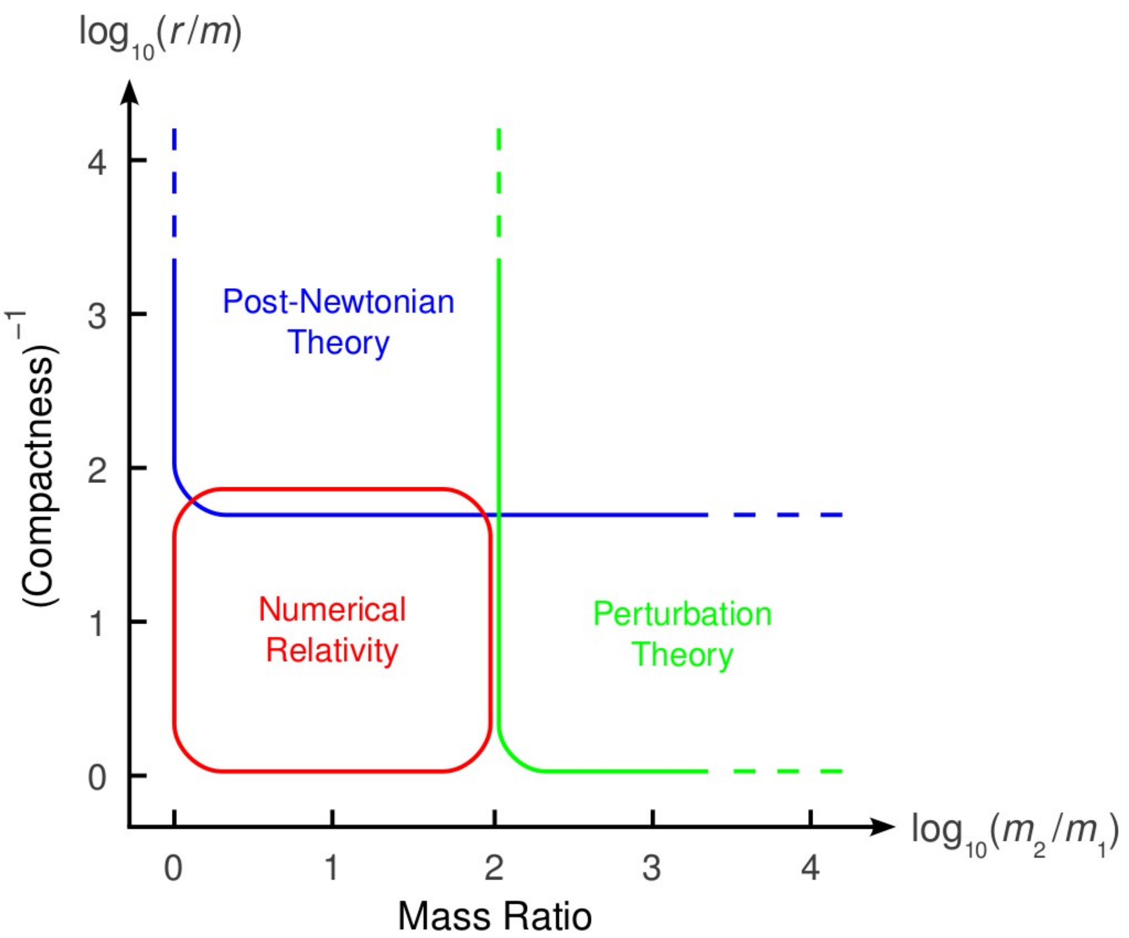
$$h \sim 4\times 10^{-21}$$

Theoretical waveforms

Theoretical input:

- 90s: CBC PN waveforms (Blanchet, Iyer, Damour, Deruelle, Will, Wiseman, ...)
- 00s: CBC Effective One Body "EOB" (Damour, Buonanno)
- 06: BBH numerical simulation (Pretorius, Baker, Loustos, Campanelli)





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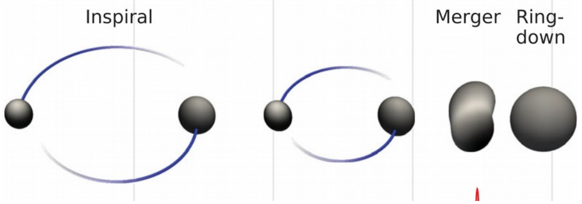
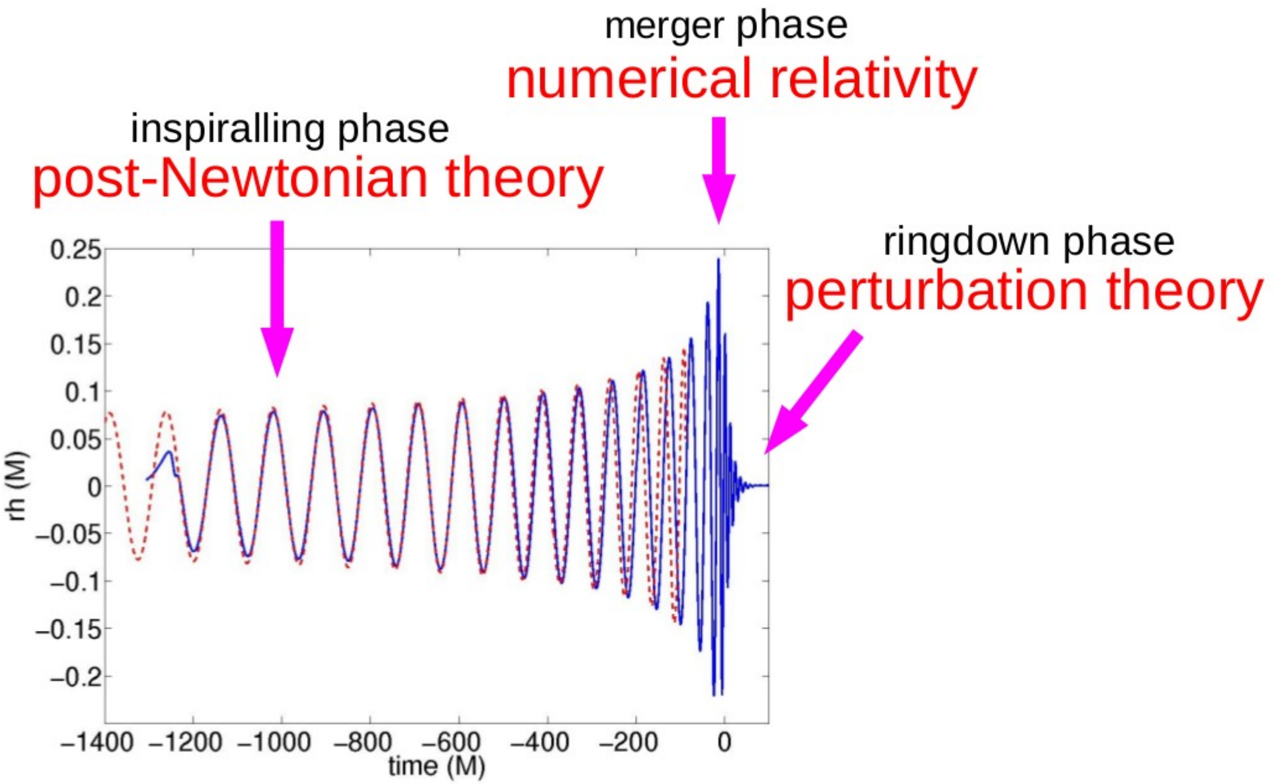




Inspiral phase: the phase is driven by the "chirp" mass

$$M_{chirp}=\frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}=\frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3}f^{-11/3}\dot{f}\right]^{3/5}$$

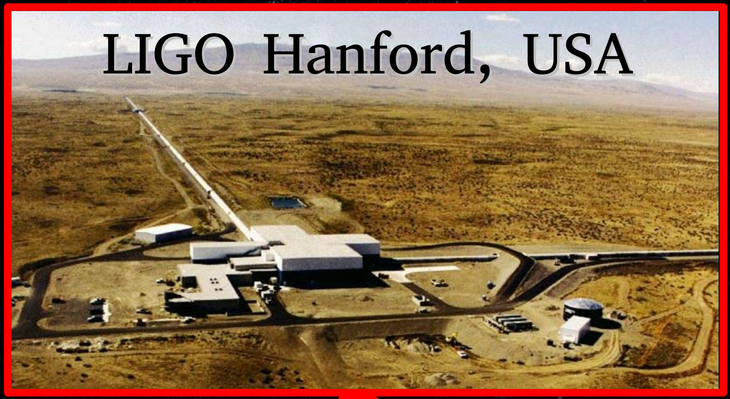
➙ Input for GW searches
➙ Input for parameter estimation analyses

September 14, 2015:

A gravitational-wave event is detected for the first time in ground-based detectors

GW150914

Ground-based gravitational-wave detectors


LIGO Hanford, USA


LIGO Hanford, USA
LIGO Livingston, USA


LIGO Hanford, USA
Virgo Pisa, Italy
LIGO Livingston, USA

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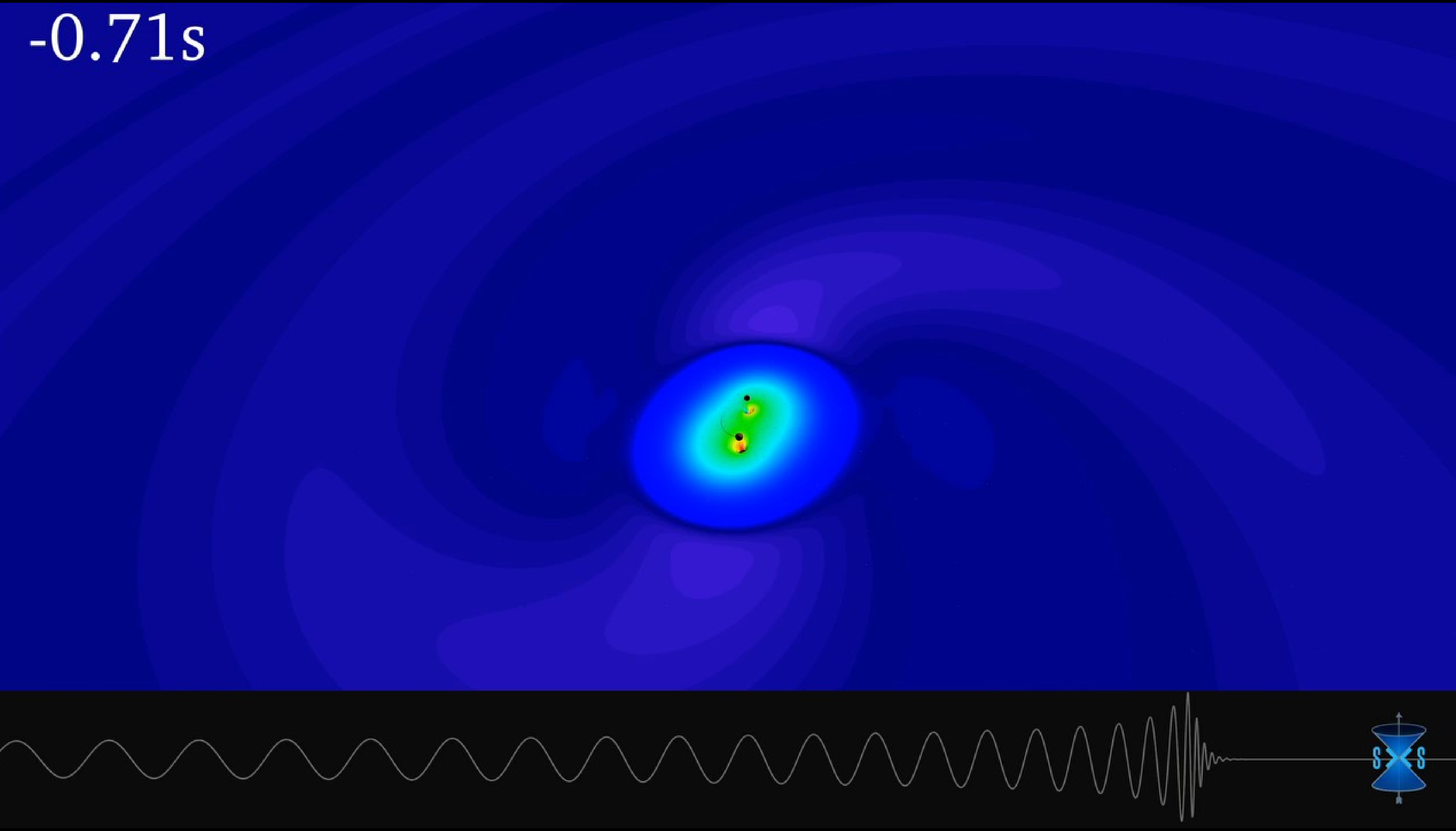
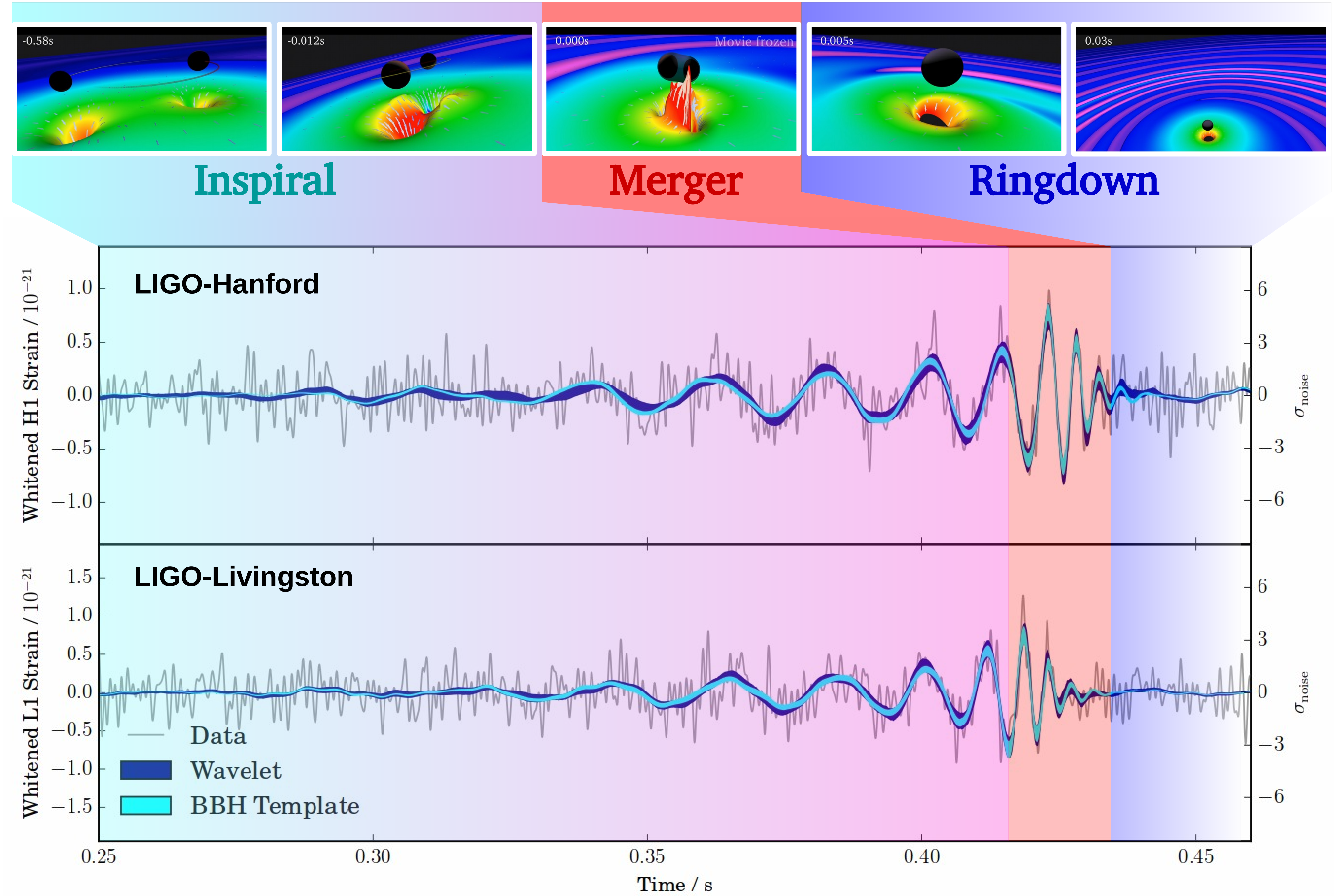
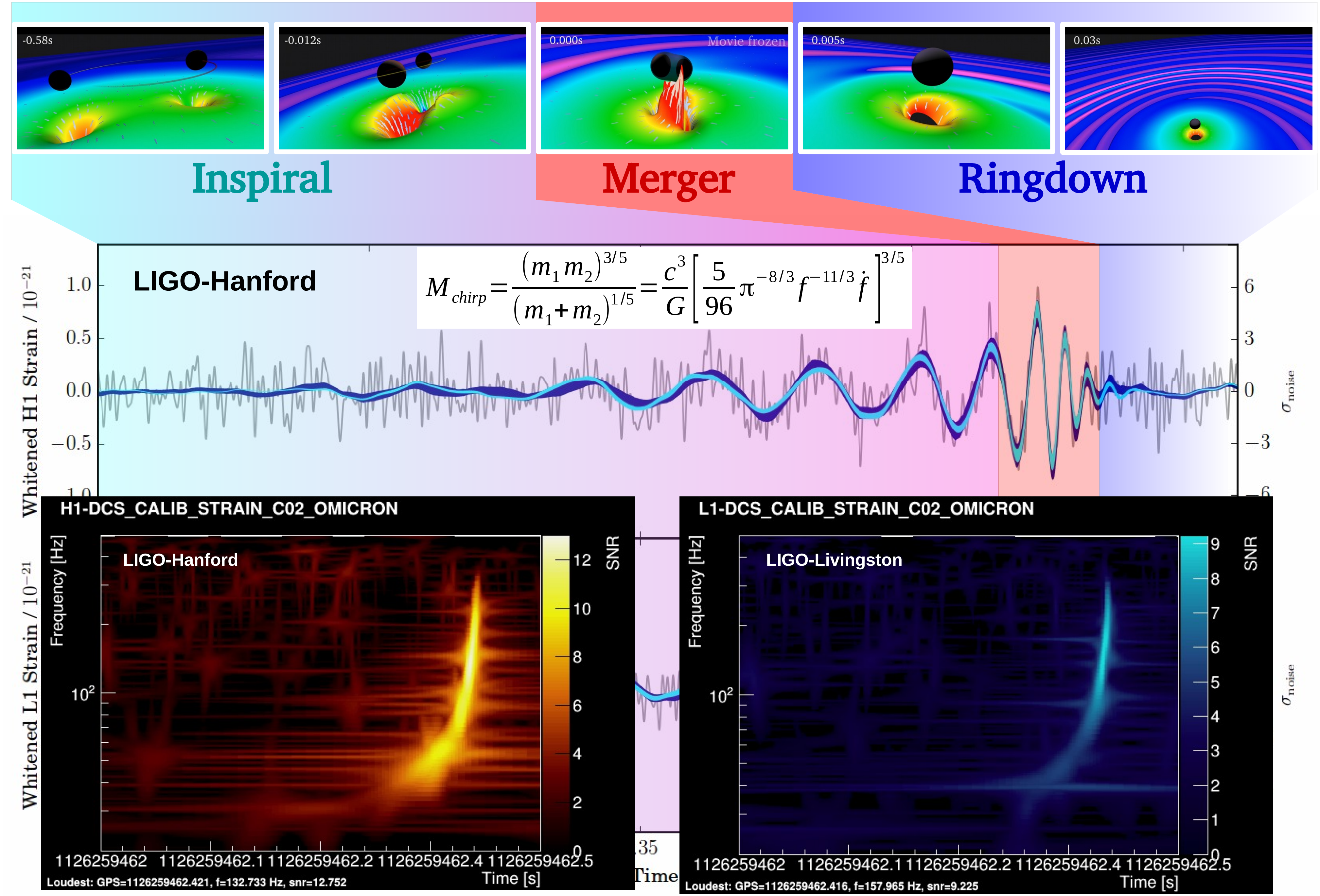




Compact Binary Coalescence = CBC



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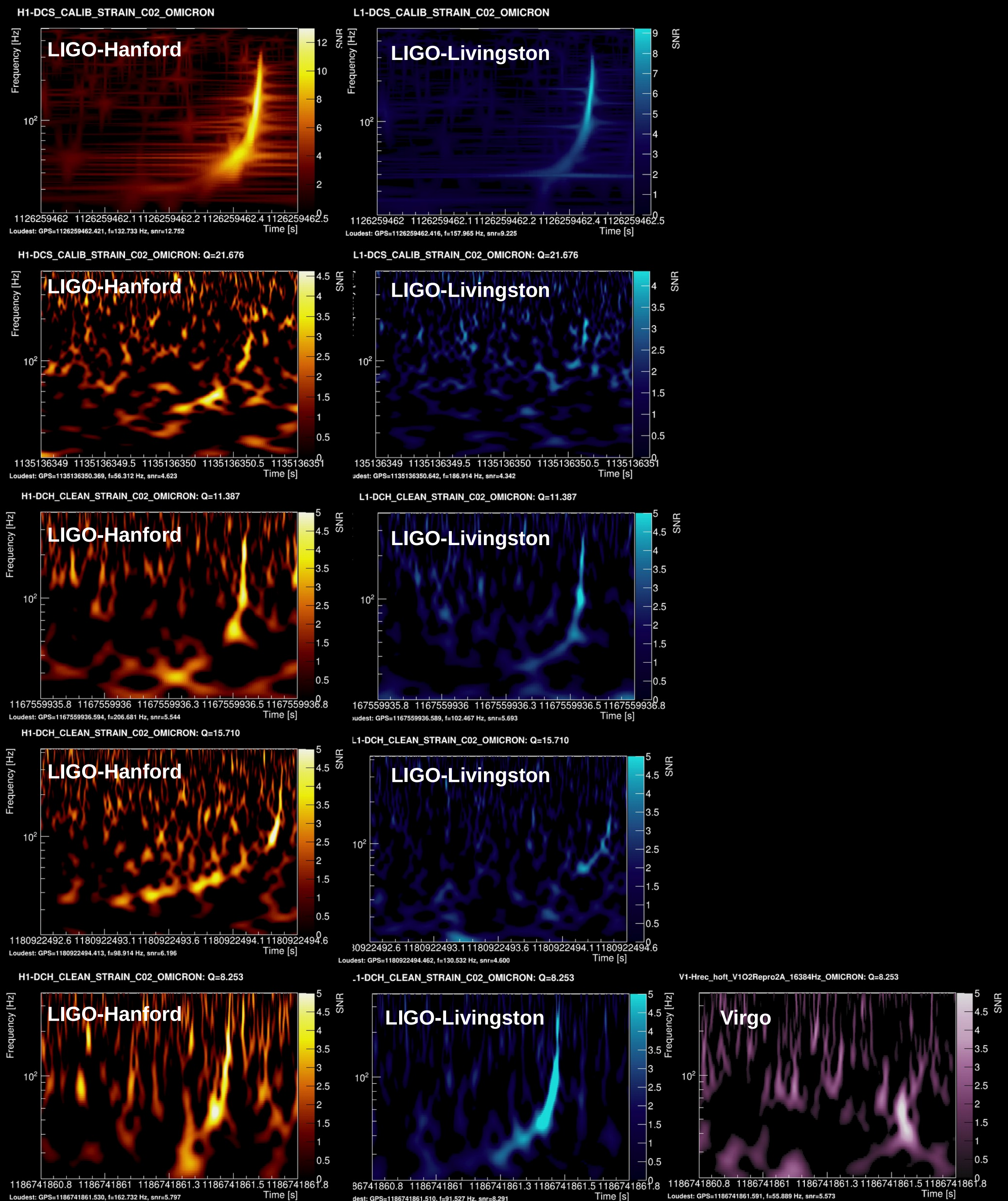


GW150914

GW151226

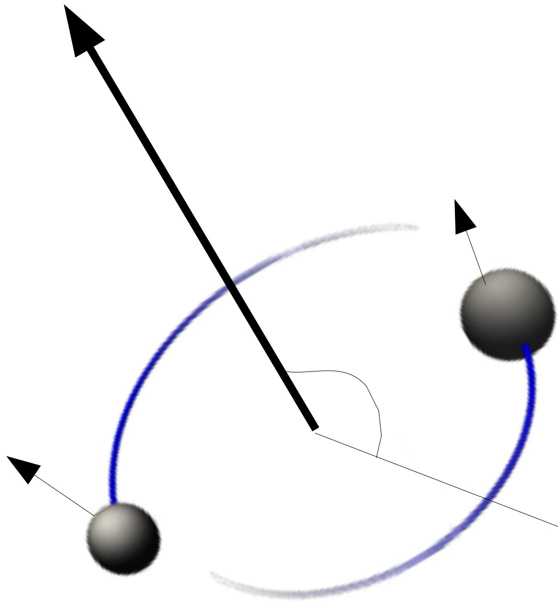
GW170104

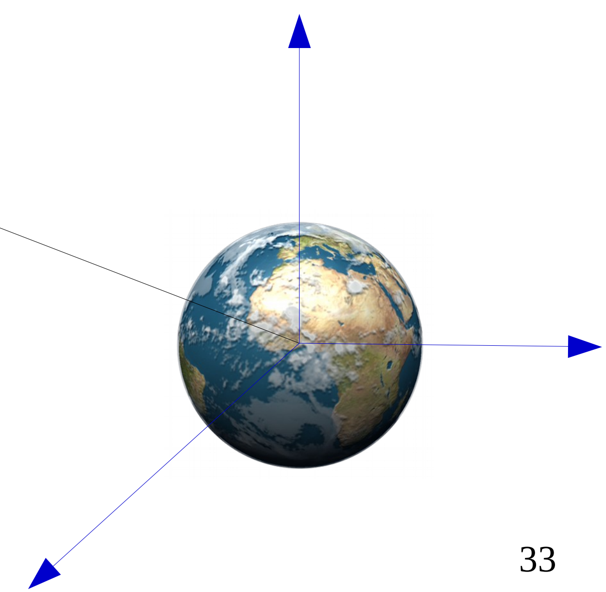
GW170608

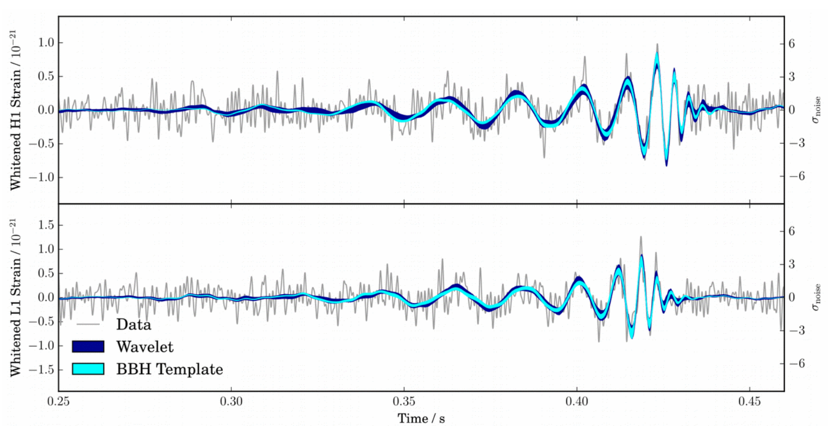
GW170814

Parameter estimation

How many parameters are needed to fully describe the system?

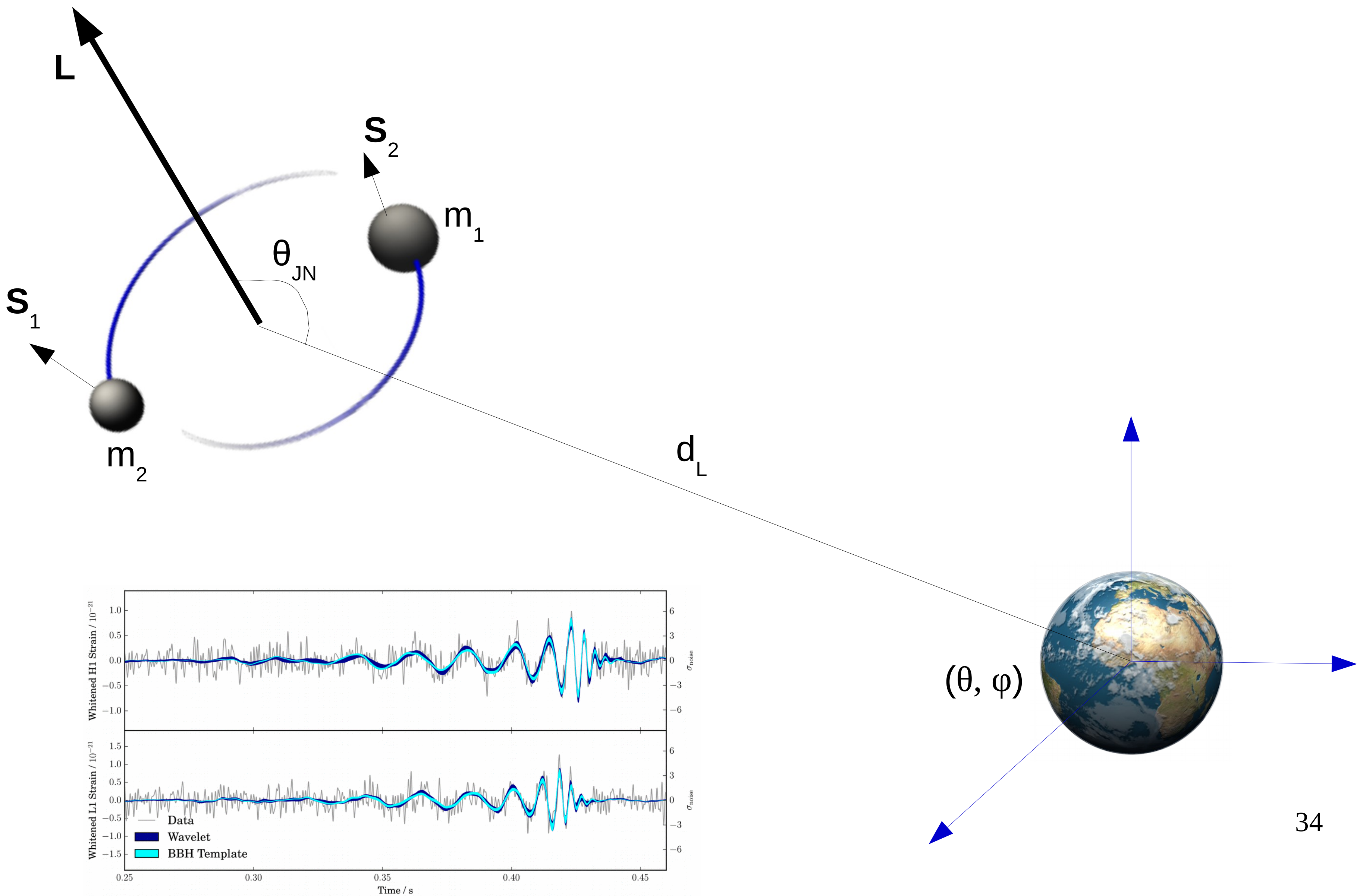






Parameter estimation

8 intrinsic parameters: masses and spins
9 extrinsic parameters: distance, position (x2), orientation (x2), orbital ellipticity (x2), coalescence time and phase)



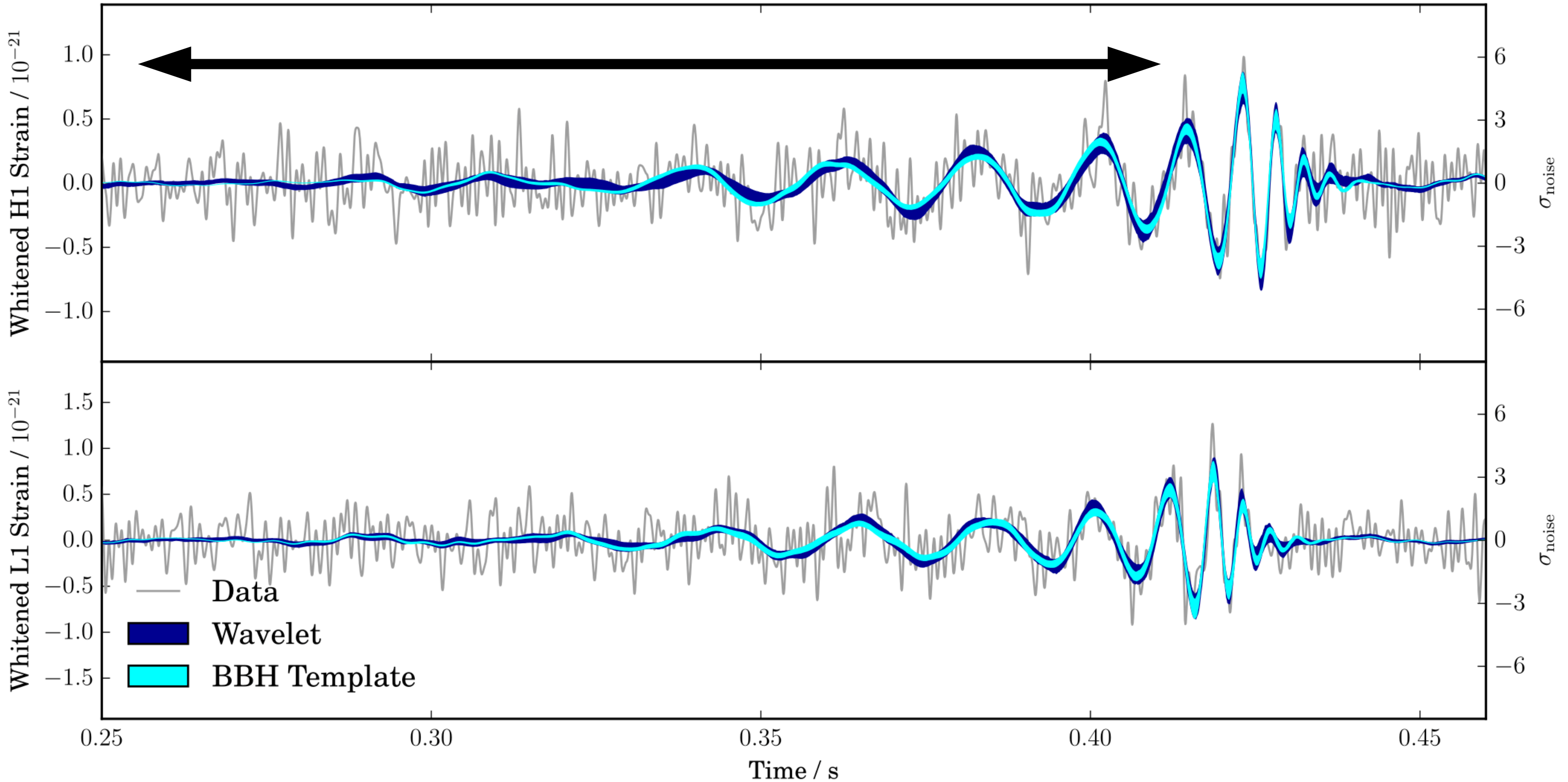
Parameter estimation

Inspiral phase: PN perturbative expansion (v/c)

Leading order → phase evolution driven by the chirp mass (tight constraints)

Next order → m2/m1 and spins **// L**

Next orders → full spins

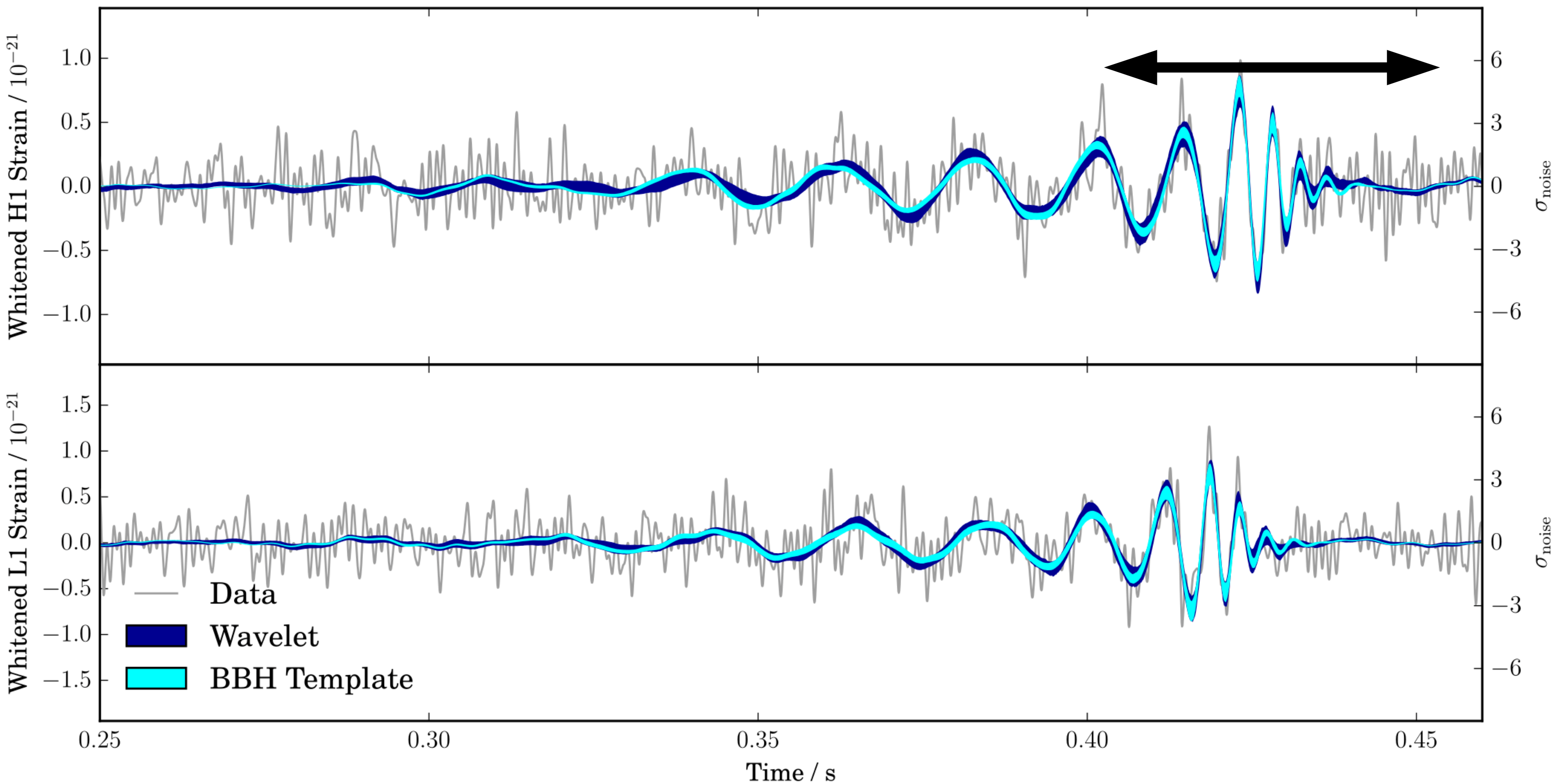


Parameter estimation

Late inspiral – merger – ringdown: numerical relativity waveforms

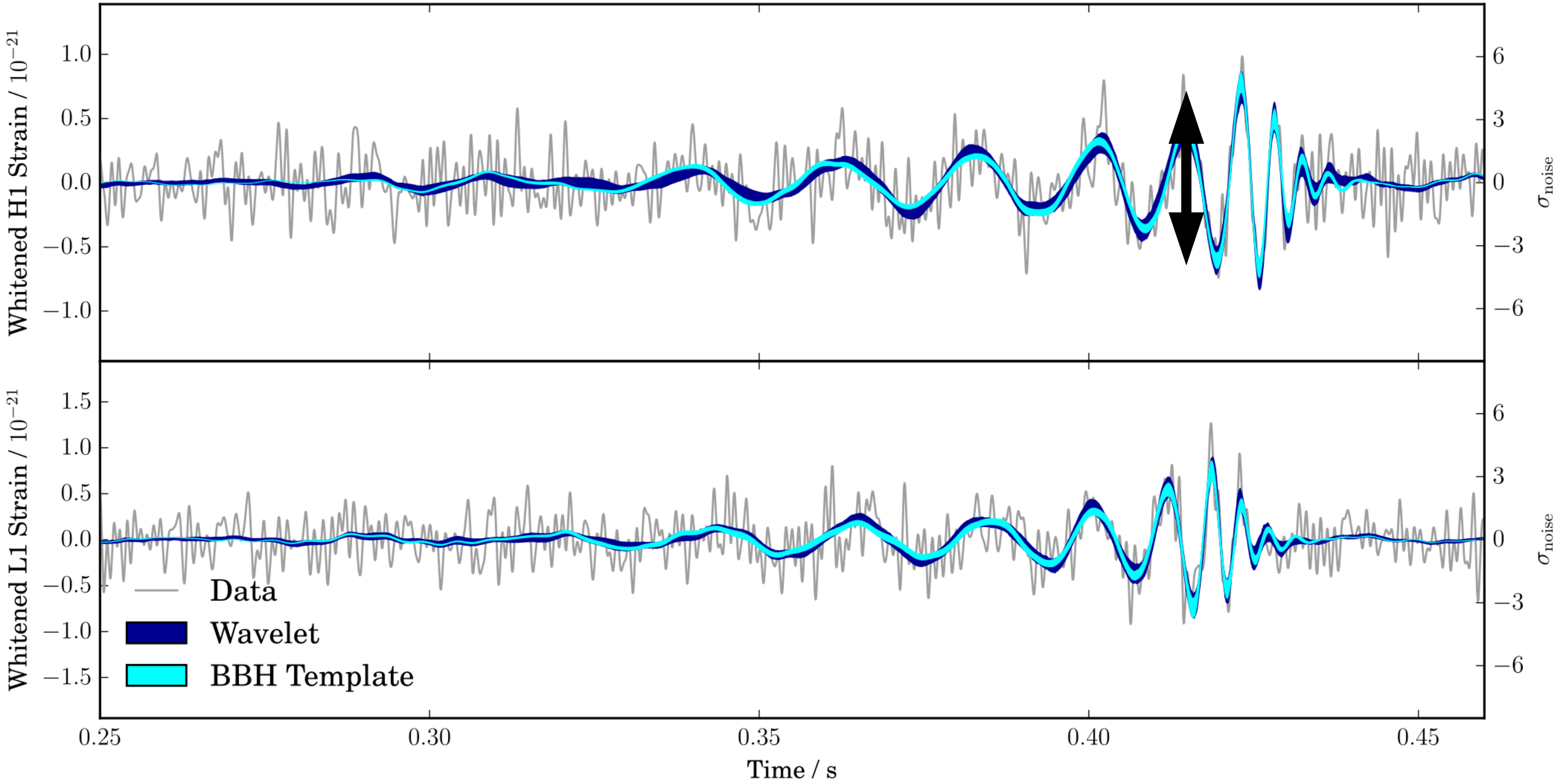
Late inspiral → total mass (+chirp mass + m1/m2) → individual masses

Ringdown → final BH mass and spin



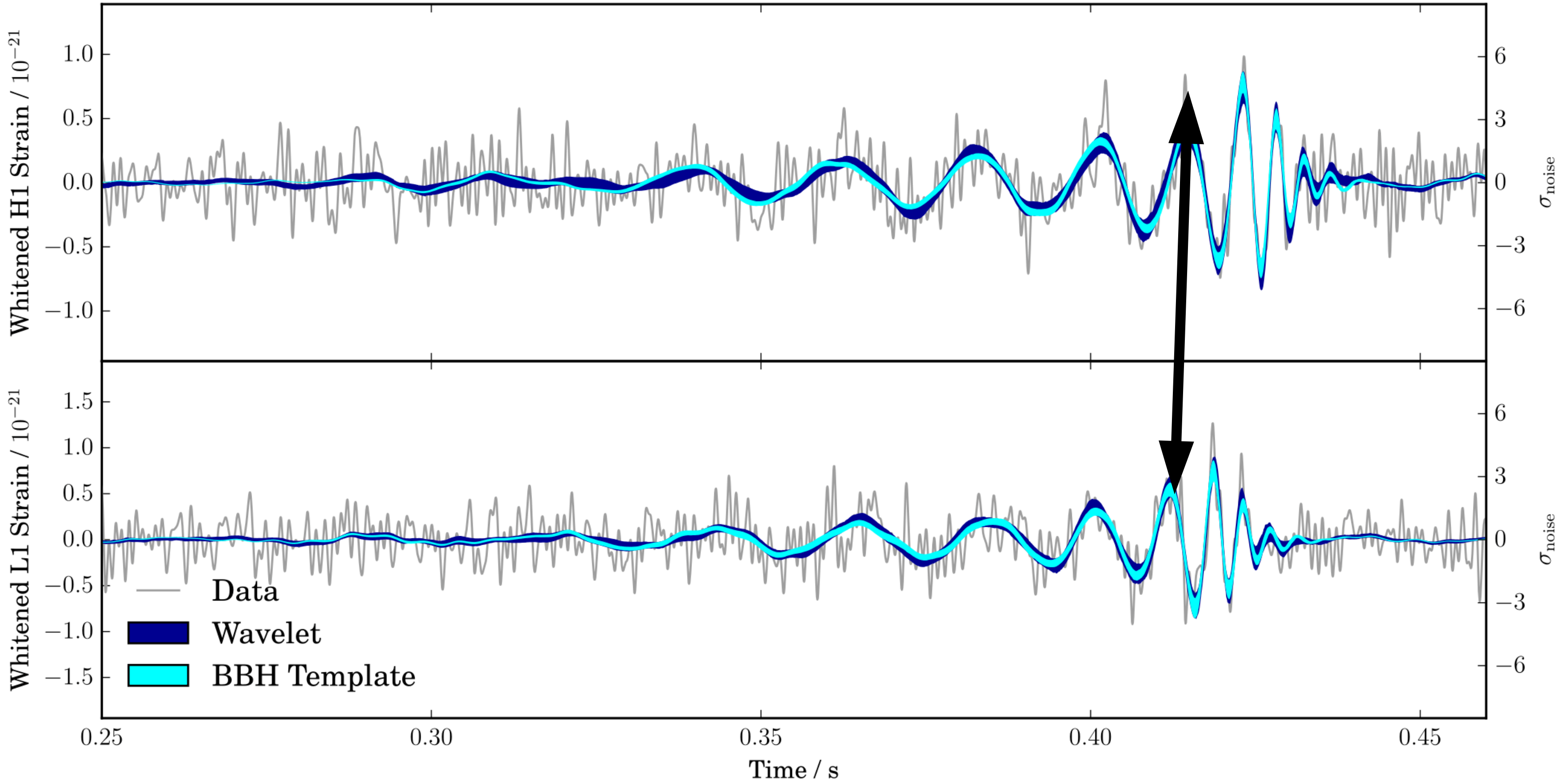
Parameter estimation

Amplitude: inversely proportional to the distance

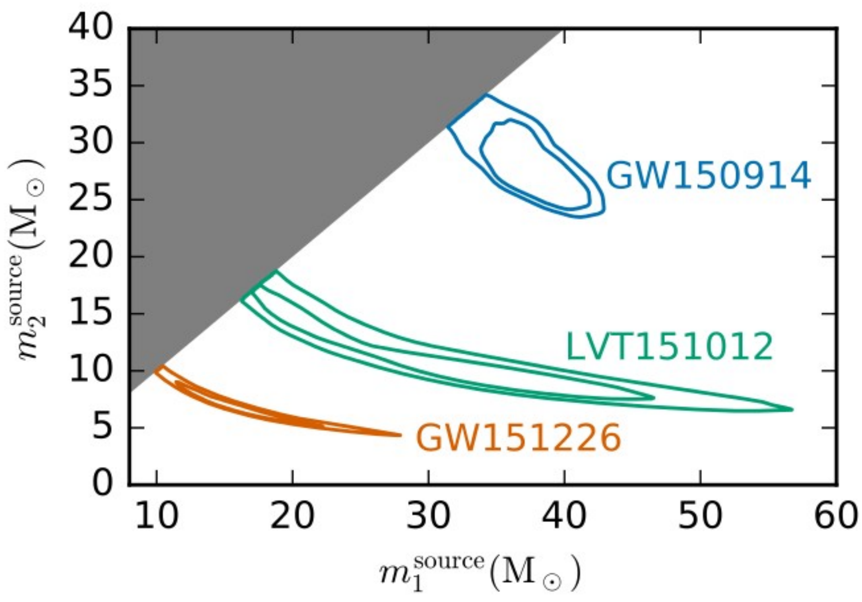


Parameter estimation

Amplitude and phase difference between sites → sky location
+ Amplitude and phase consistency



Parameter estimation



Mostly sensitive to the chirp mass
→ m_1, m_2 degeneracy

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

GW150914

$$m_1 = 36.2^{+5.2}_{-3.2}\,M_{sun}$$

$$m_2 = 29.1^{+3.7}_{-4.4}\,M_{sun}$$

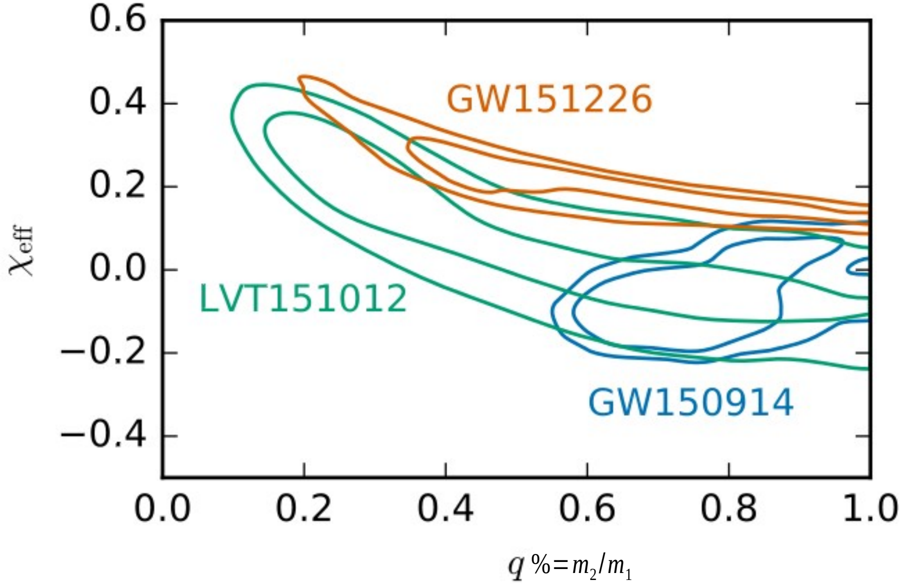
GW151226

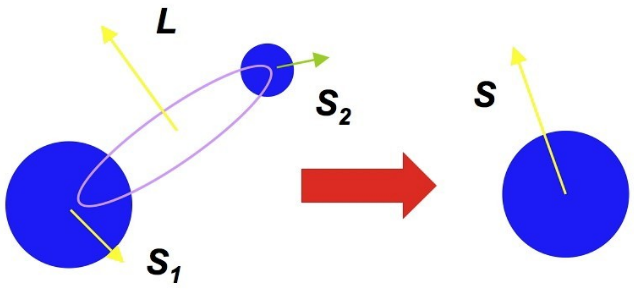
$$m_1 = 14.2^{+8.3}_{-3.7}\,M_{sun}$$

$$m_2 = 7.5^{+2.3}_{-2.3}\,M_{sun}$$

→ All the components are black holes
→ Very high masses for GW150914

Parameter estimation





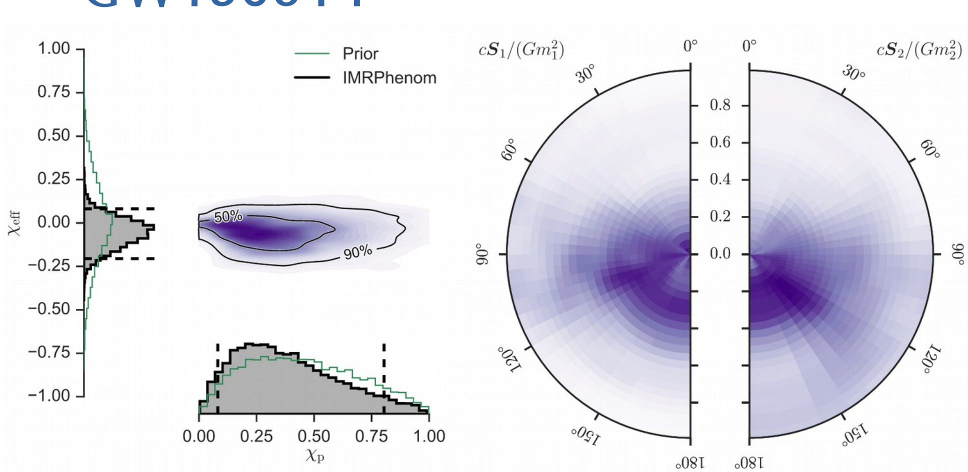
$$\chi_{eff} = \frac{m_1 a_{1z} + m_2 a_{2z}}{m_1 + m_2}$$

→ not well constrained

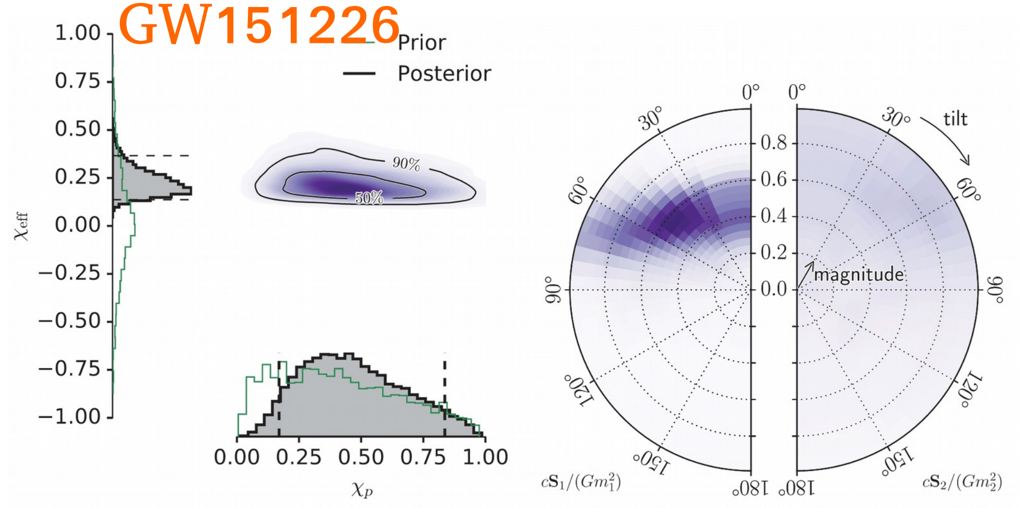
GW151226: at least one black hole is a Kerr black hole
spin >0.2

Uninformative about precession

GW150914



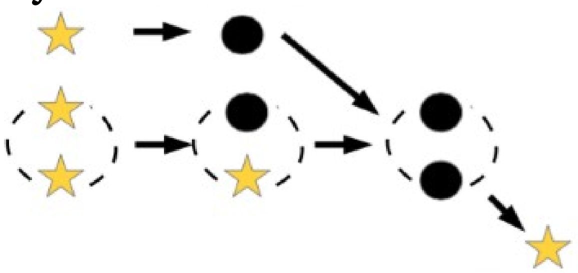
GW151226



BBH formation

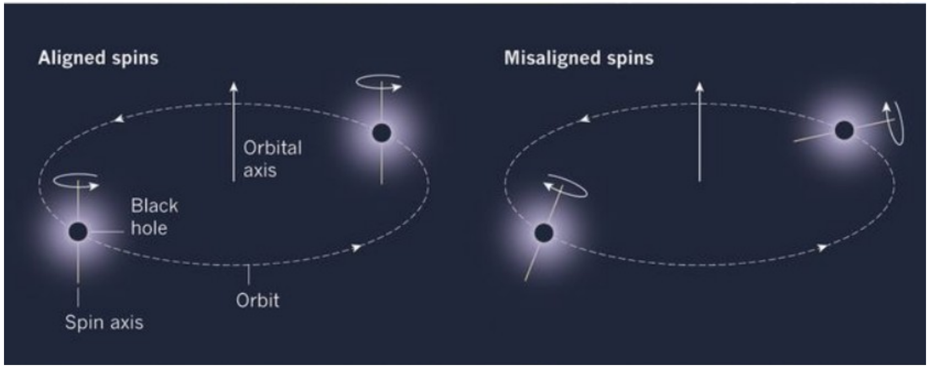
Isolated binary in galactic fields

Dynamical interactions in clusters



How can we discriminate these 2 scenarios?

→ spins!



Isolated binary:

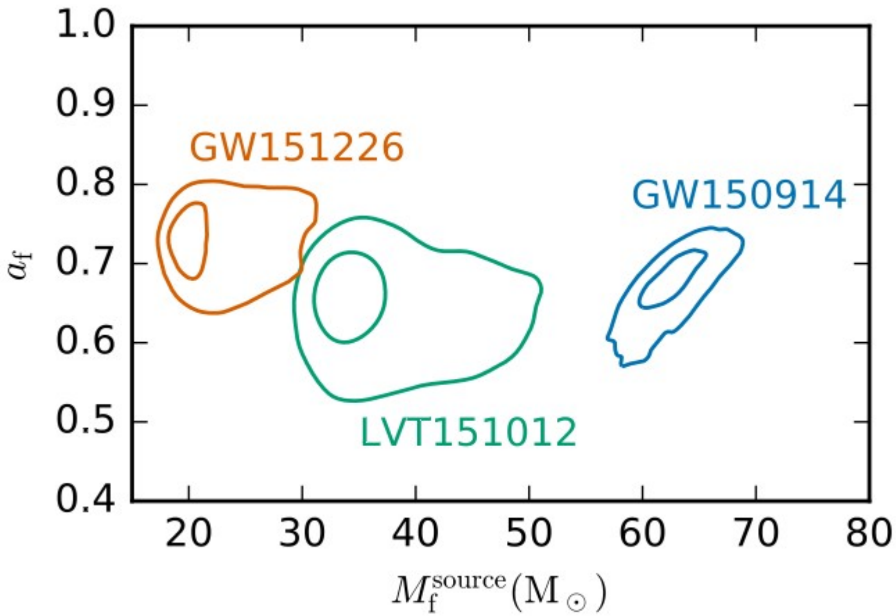
Spins preferentially aligned with the binary orbital angular momentum

Cluster binary:

Isotropic spin orientations

Parameter estimation

Final mass & spin



GW150914

$$M_f = 62.3^{+3.7}_{-3.1} M_{sun}$$

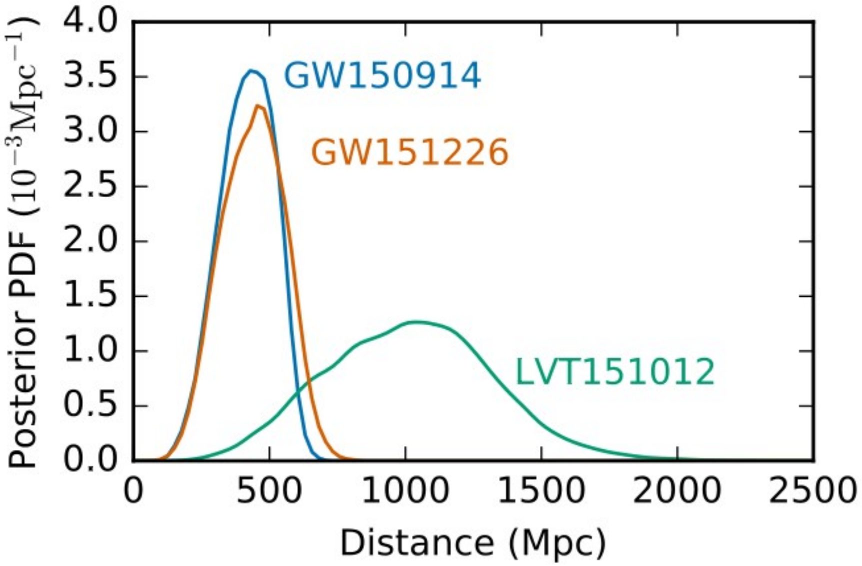
$$a_f = 0.68^{+0.05}_{-0.06}$$

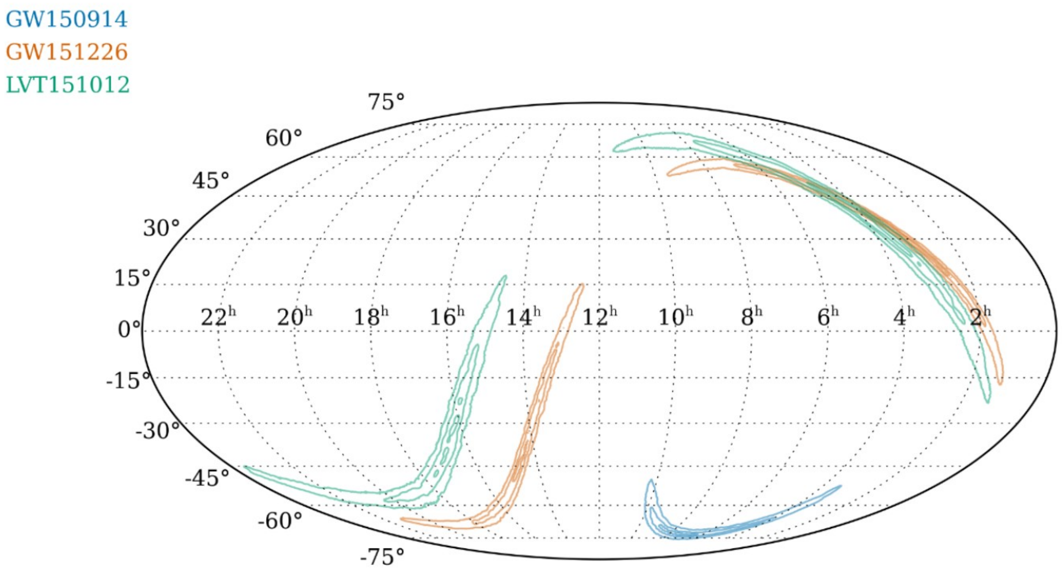
GW151226

$$M_f = 20.8^{+6.1}_{-1.7} M_{sun}$$

$$a_f = 0.74^{+0.06}_{-0.06}$$

Parameter estimation





90% credible region for sky location:

→ **GW150914 = 230** deg²

→ **GW151226 = 850** deg²

Limited accuracy with **2** detectors

GW150914

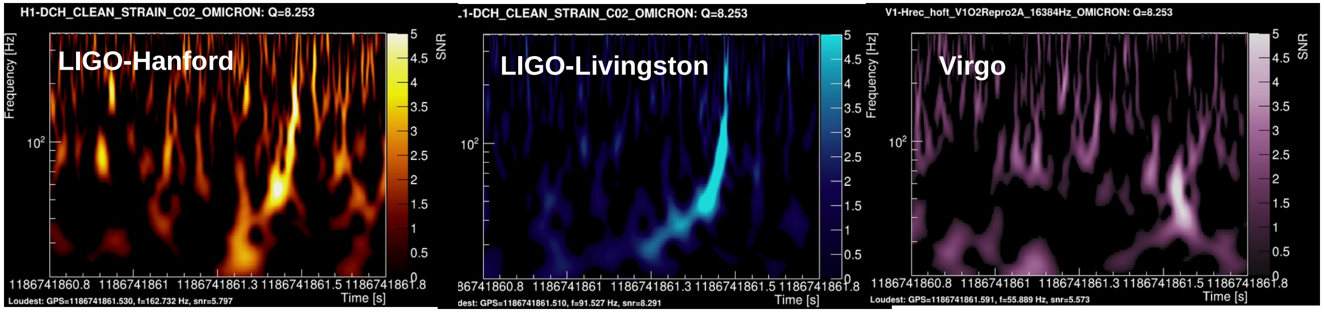
$$D_L = 420^{+150}_{-180}\,Mpc \qquad z = 0.09^{+0.03}_{-0.04}$$

GW151226

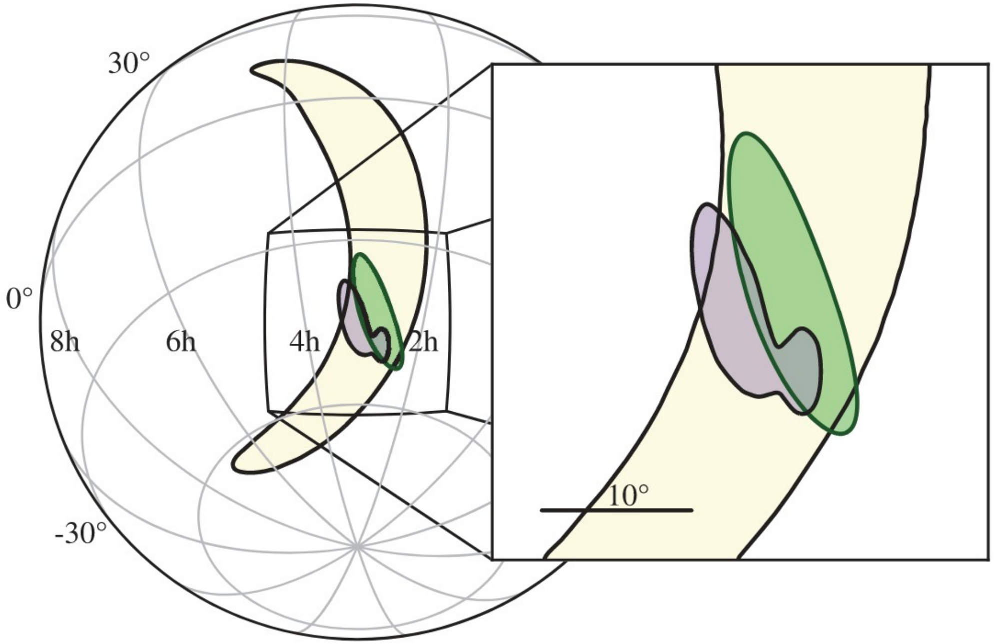
$$D_L = 440^{+180}_{-190}\,Mpc \qquad z = 0.09^{+0.03}_{-0.04}$$

(Lambda-CDM cosmology)

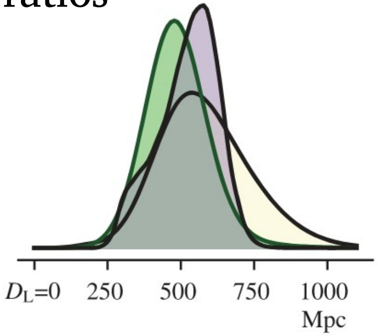
Parameter estimation



GW170814



3 detectors → triangulation using time differences, phase differences and amplitude ratios



~**1000** deg² (LIGO)

~**60** deg² (LIGO**+**Virgo)

Luminosity distance **=** 540^{+130}_{-210} Mpc

Parameter estimation

Event	$m_1/M_\odot$	$m_2/M_\odot$	$\mathcal{M}/M_\odot$	χ_eff	$M_f/M_\odot$	a_f	$E_\mathrm{rad}/(M_\odot c^2)$	$\ell_\mathrm{peak}/(\mathrm{erg\,s^{-1}})$	d_L/Mpc	z	$\Delta\Omega/\mathrm{deg}^2$
GW150914	$35.6^{+4.7}_{-3.1}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.7}_{-1.5}$	$-0.01^{+0.12}_{-0.13}$	$63.1^{+3.4}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	$3.1^{+0.4}_{-0.4}$	$3.6^{+0.4}_{-0.4} \times 10^{56}$	440^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$	182
GW151012	$23.2^{+14.9}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.1}_{-1.2}$	$0.05^{+0.31}_{-0.20}$	$35.6^{+10.8}_{-3.8}$	$0.67^{+0.13}_{-0.11}$	$1.6^{+0.6}_{-0.5}$	$3.2^{+0.8}_{-1.7} \times 10^{56}$	1080^{+550}_{-490}	$0.21^{+0.09}_{-0.09}$	1523
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.5}$	$8.9^{+0.3}_{-0.3}$	$0.18^{+0.20}_{-0.12}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$	$1.0^{+0.1}_{-0.2}$	$3.4^{+0.7}_{-1.7} \times 10^{56}$	450^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$	1033
GW170104	$30.8^{+7.3}_{-5.6}$	$20.0^{+4.9}_{-4.6}$	$21.4^{+2.2}_{-1.8}$	$-0.04^{+0.17}_{-0.21}$	$48.9^{+5.1}_{-4.0}$	$0.66^{+0.08}_{-0.11}$	$2.2^{+0.5}_{-0.5}$	$3.3^{+0.6}_{-1.0} \times 10^{56}$	990^{+440}_{-430}	$0.20^{+0.08}_{-0.08}$	921
GW170608	$11.0^{+5.5}_{-1.7}$	$7.6^{+1.4}_{-2.2}$	$7.9^{+0.2}_{-0.2}$	$0.03^{+0.19}_{-0.07}$	$17.8^{+3.4}_{-0.7}$	$0.69^{+0.04}_{-0.04}$	$0.9^{+0.0}_{-0.1}$	$3.5^{+0.4}_{-1.3} \times 10^{56}$	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$	392
GW170729	$50.2^{+16.2}_{-10.2}$	$34.0^{+9.1}_{-10.1}$	$35.4^{+6.5}_{-4.8}$	$0.37^{+0.21}_{-0.25}$	$79.5^{+14.7}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	$4.8^{+1.7}_{-1.7}$	$4.2^{+0.9}_{-1.5} \times 10^{56}$	2840^{+1400}_{-1360}	$0.49^{+0.19}_{-0.21}$	1041
GW170809	$35.0^{+8.3}_{-5.9}$	$23.8^{+5.1}_{-5.2}$	$24.9^{+2.1}_{-1.7}$	$0.08^{+0.17}_{-0.17}$	$56.3^{+5.2}_{-3.8}$	$0.70^{+0.08}_{-0.09}$	$2.7^{+0.6}_{-0.6}$	$3.5^{+0.6}_{-0.9} \times 10^{56}$	1030^{+320}_{-390}	$0.20^{+0.05}_{-0.07}$	308
GW170814	$30.6^{+5.6}_{-3.0}$	$25.2^{+2.8}_{-4.0}$	$24.1^{+1.4}_{-1.1}$	$0.07^{+0.12}_{-0.12}$	$53.2^{+3.2}_{-2.4}$	$0.72^{+0.07}_{-0.05}$	$2.7^{+0.4}_{-0.3}$	$3.7^{+0.4}_{-0.5} \times 10^{56}$	600^{+150}_{-220}	$0.12^{+0.03}_{-0.04}$	87
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$	$0.00^{+0.02}_{-0.01}$	≤ 2.8	≤ 0.89	≥ 0.04	$\geq 0.1 \times 10^{56}$	40^{+7}_{-15}	$0.01^{+0.00}_{-0.00}$	16
GW170818	$35.4^{+7.5}_{-4.7}$	$26.7^{+4.3}_{-5.2}$	$26.5^{+2.1}_{-1.7}$	$-0.09^{+0.18}_{-0.21}$	$59.4^{+4.9}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	$2.7^{+0.5}_{-0.5}$	$3.4^{+0.5}_{-0.7} \times 10^{56}$	1060^{+420}_{-380}	$0.21^{+0.07}_{-0.07}$	39
GW170823	$39.5^{+11.2}_{-6.7}$	$29.0^{+6.7}_{-7.8}$	$29.2^{+4.6}_{-3.6}$	$0.09^{+0.22}_{-0.26}$	$65.4^{+10.1}_{-7.4}$	$0.72^{+0.09}_{-0.12}$	$3.3^{+1.0}_{-0.9}$	$3.6^{+0.7}_{-1.1} \times 10^{56}$	1940^{+970}_{-900}	$0.35^{+0.15}_{-0.15}$	1666

Parameter estimation

Event	$m_1/M_\odot$	$m_2/M_\odot$	$\mathcal{M}/M_\odot$	$\chi_{\rm eff}$	$M_f/M_\odot$	a_f	$E_{\rm rad}/(M_\odot c^2)$	$\ell_{\rm peak}/({\rm erg\,s^{-1}})$	$d_L/{\rm Mpc}$	z	$\Delta\Omega/{\rm deg}^2$
GW150914	$35.6^{+4.7}_{-3.1}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.7}_{-1.5}$	$-0.01^{+0.12}_{-0.13}$	$63.1^{+3.4}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	$3.1^{+0.4}_{-0.4}$	$3.6^{+0.4}_{-0.4}\times10^{56}$	440^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$	182
GW151012	$23.2^{+14.9}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.1}_{-1.2}$	$0.05^{+0.31}_{-0.20}$	$35.6^{+10.8}_{-3.8}$	$0.67^{+0.13}_{-0.11}$	$1.6^{+0.6}_{-0.5}$	$3.2^{+0.8}_{-1.7}\times10^{56}$	1080^{+550}_{-490}	$0.21^{+0.09}_{-0.09}$	1523
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.5}$	$8.9^{+0.3}_{-0.3}$	$0.18^{+0.20}_{-0.12}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$	$1.0^{+0.1}_{-0.2}$	$3.4^{+0.7}_{-1.7}\times10^{56}$	450^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$	1033
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GW170608	$11.0^{+5.5}_{-1.7}$	$7.6^{+1.4}_{-2.2}$	$7.9^{+0.2}_{-0.2}$	$0.03^{+0.19}_{-0.07}$	$17.8^{+3.4}_{-0.7}$	$0.69^{+0.04}_{-0.04}$	$0.9^{+0.0}_{-0.1}$	$3.5^{+0.4}_{-1.3}\times10^{56}$	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$	392
GW170729	$50.2^{+16.2}_{-10.2}$	$34.0^{+9.1}_{-10.1}$	$35.4^{+6.5}_{-4.8}$	$0.37^{+0.21}_{-0.25}$	$79.5^{+14.7}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	$4.8^{+1.7}_{-1.7}$	$4.2^{+0.9}_{-1.5}\times10^{56}$	2840^{+1400}_{-1360}	$0.49^{+0.19}_{-0.21}$	1041
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GW170818	$35.4^{+7.5}_{-4.7}$	$26.7^{+4.3}_{-5.2}$	$26.5^{+2.1}_{-1.7}$	$-0.09^{+0.18}_{-0.21}$	$59.4^{+4.9}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	$2.7^{+0.5}_{-0.5}$	$3.4^{+0.5}_{-0.7}\times10^{56}$	1060^{+420}_{-380}	$0.21^{+0.07}_{-0.07}$	39
GW170823	$39.5^{+11.2}_{-6.7}$	$29.0^{+6.7}_{-7.8}$	$29.2^{+4.6}_{-3.6}$	$0.09^{+0.22}_{-0.26}$	$65.4^{+10.1}_{-7.4}$	$0.72^{+0.09}_{-0.12}$	$3.3^{+1.0}_{-0.9}$	$3.6^{+0.7}_{-1.1}\times10^{56}$	1940^{+970}_{-900}	$0.35^{+0.15}_{-0.15}$	1666

Testing General Relativity

Modified dispersion relation (ex: LIV theories): $E^2 = p^2 c^2 + A^\alpha c^\alpha$

- massive graviton: $\alpha = 0$
- multifractal theories: $\alpha = 2.5$
- doubly special relativity: $\alpha = 3$
- extra-dimensions: $\alpha = 4$

→ modified propagation velocity: $\dfrac{v_g}{c} = 1 + (\alpha - 1)\dfrac{AE^{\alpha-2}}{2}$

Parameter estimation

Event	$m_1/M_\odot$	$m_2/M_\odot$	$\mathcal{M}/M_\odot$	$\chi_{\rm eff}$	$M_f/M_\odot$	a_f	$E_{\rm rad}/(M_\odot c^2)$	$\ell_{\rm peak}/({\rm erg\,s^{-1}})$	d_L/Mpc	z	$\Delta\Omega/{\rm deg}^2$
GW150914	$35.6^{+4.7}_{-3.1}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.7}_{-1.5}$	$-0.01^{+0.12}_{-0.13}$	$63.1^{+3.4}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	$3.1^{+0.4}_{-0.4}$	$3.6^{+0.4}_{-0.4}\times10^{56}$	440^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$	182
GW151012	$23.2^{+14.9}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	$15.2^{+2.1}_{-1.2}$	$0.05^{+0.31}_{-0.20}$	$35.6^{+10.8}_{-3.8}$	$0.67^{+0.13}_{-0.11}$	$1.6^{+0.6}_{-0.5}$	$3.2^{+0.8}_{-1.7}\times10^{56}$	1080^{+550}_{-490}	$0.21^{+0.09}_{-0.09}$	1523
GW151226	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.5}$	$8.9^{+0.3}_{-0.3}$	$0.18^{+0.20}_{-0.12}$	$20.5^{+6.4}_{-1.5}$	$0.74^{+0.07}_{-0.05}$	$1.0^{+0.1}_{-0.2}$	$3.4^{+0.7}_{-1.7}\times10^{56}$	450^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$	1033
GW170104	$30.8^{+7.3}_{-5.6}$	$20.0^{+4.9}_{-4.6}$	$21.4^{+2.2}_{-1.8}$	$-0.04^{+0.17}_{-0.21}$	$48.9^{+5.1}_{-4.0}$	$0.66^{+0.08}_{-0.11}$	$2.2^{+0.5}_{-0.5}$	$3.3^{+0.6}_{-1.0}\times10^{56}$	990^{+440}_{-430}	$0.20^{+0.08}_{-0.08}$	921
GW170608	$11.0^{+5.5}_{-1.7}$	$7.6^{+1.4}_{-2.2}$	$7.9^{+0.2}_{-0.2}$	$0.03^{+0.19}_{-0.07}$	$17.8^{+3.4}_{-0.7}$	$0.69^{+0.04}_{-0.04}$	$0.9^{+0.0}_{-0.1}$	$3.5^{+0.4}_{-1.3}\times10^{56}$	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$	392
GW170729	$50.2^{+16.2}_{-10.2}$	$34.0^{+9.1}_{-10.1}$	$35.4^{+6.5}_{-4.8}$	$0.37^{+0.21}_{-0.25}$	$79.5^{+14.7}_{-10.2}$	$0.81^{+0.07}_{-0.13}$	$4.8^{+1.7}_{-1.7}$	$4.2^{+0.9}_{-1.5}\times10^{56}$	2840^{+1400}_{-1360}	$0.49^{+0.19}_{-0.21}$	1041
GW170809	$35.0^{+8.3}_{-5.9}$	$23.8^{+5.1}_{-5.2}$	$24.9^{+2.1}_{-1.7}$	$0.08^{+0.17}_{-0.17}$	$56.3^{+5.2}_{-3.8}$	$0.70^{+0.08}_{-0.09}$	$2.7^{+0.6}_{-0.6}$	$3.5^{+0.6}_{-0.9}\times10^{56}$	1030^{+320}_{-390}	$0.20^{+0.05}_{-0.07}$	308
GW170814	$30.6^{+5.6}_{-3.0}$	$25.2^{+2.8}_{-4.0}$	$24.1^{+1.4}_{-1.1}$	$0.07^{+0.12}_{-0.12}$	$53.2^{+3.2}_{-2.4}$	$0.72^{+0.07}_{-0.05}$	$2.7^{+0.4}_{-0.3}$	$3.7^{+0.4}_{-0.5}\times10^{56}$	600^{+150}_{-220}	$0.12^{+0.03}_{-0.04}$	87
GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$1.186^{+0.001}_{-0.001}$	$0.00^{+0.02}_{-0.01}$	≤ 2.8	≤ 0.89	≥ 0.04	$\geq 0.1\times10^{56}$	40^{+7}_{-15}	$0.01^{+0.00}_{-0.00}$	16
GW170818	$35.4^{+7.5}_{-4.7}$	$26.7^{+4.3}_{-5.2}$	$26.5^{+2.1}_{-1.7}$	$-0.09^{+0.18}_{-0.21}$	$59.4^{+4.9}_{-3.8}$	$0.67^{+0.07}_{-0.08}$	$2.7^{+0.5}_{-0.5}$	$3.4^{+0.5}_{-0.7}\times10^{56}$	1060^{+420}_{-380}	$0.21^{+0.07}_{-0.07}$	39
GW170823	$39.5^{+11.2}_{-6.7}$	$29.0^{+6.7}_{-7.8}$	$29.2^{+4.6}_{-3.6}$	$0.09^{+0.22}_{-0.26}$	$65.4^{+10.1}_{-7.4}$	$0.72^{+0.09}_{-0.12}$	$3.3^{+1.0}_{-0.9}$	$3.6^{+0.7}_{-1.1}\times10^{56}$	1940^{+970}_{-900}	$0.35^{+0.15}_{-0.15}$	1666

Testing General Relativity

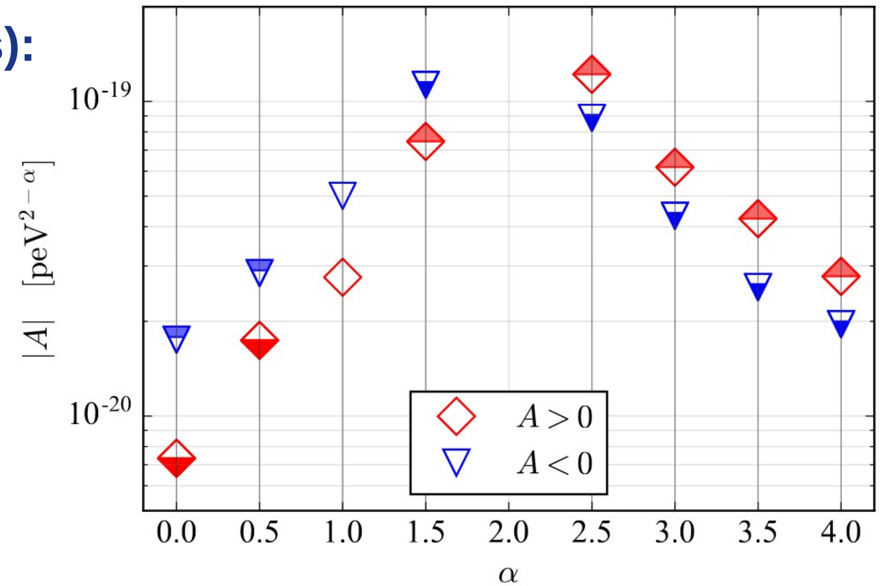
Modified dispersion relation (ex: LIV theories):

→ extra term in the evolution of the gravitational-wave phase

→ **Upper limits on A**

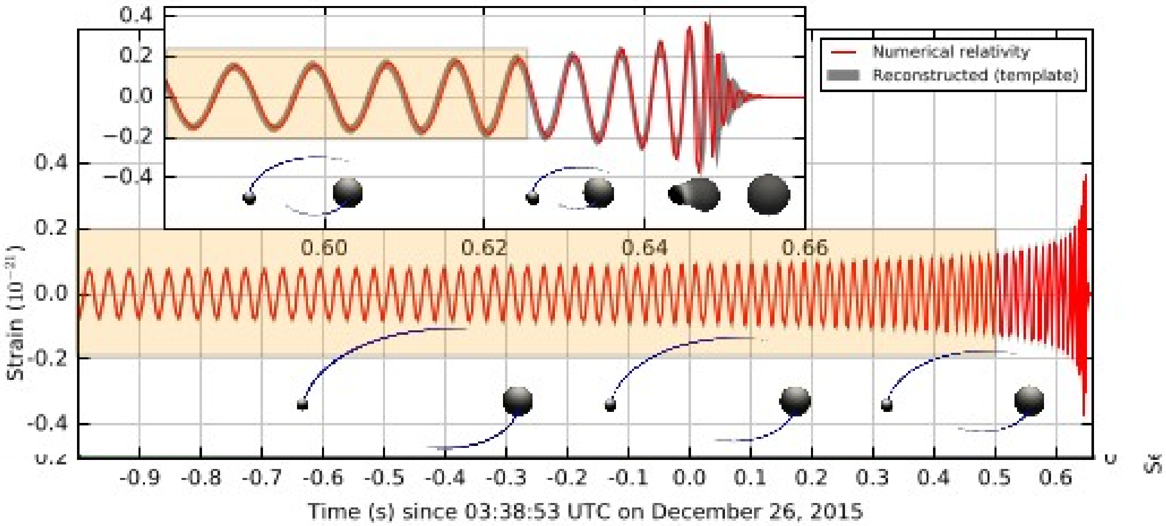
$\alpha=0\ A>0$: limit on the graviton mass:

$$m_g < 7.7\times10^{-23}\,eV/c^2$$



Parameter estimation

$E_{rad}/(M_\odot c^2)$	$\ell_{peak}/(\mathrm{erg\,s^{-1}})$	d_L/Mpc	z	$\Delta\Omega/\mathrm{deg}^2$
$^{+0.4}_{-0.4}$	$3.6^{+0.4}_{-0.4}\times 10^{56}$	440^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$	182
$^{+0.6}_{-0.5}$	$3.2^{+0.8}_{-1.7}\times 10^{56}$	1080^{+550}_{-490}	$0.21^{+0.09}_{-0.09}$	1523
$^{+0.1}_{-0.2}$	$3.4^{+0.7}_{-1.7}\times 10^{56}$	450^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$	1033
$^{+0.5}_{-0.5}$	$3.3^{+0.6}_{-1.0}\times 10^{56}$	990^{+440}_{-430}	$0.20^{+0.08}_{-0.08}$	921
$^{+0.0}_{-0.1}$	$3.5^{+0.4}_{-1.3}\times 10^{56}$	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$	392
$^{+1.7}_{-1.7}$	$4.2^{+0.9}_{-1.5}\times 10^{56}$	2840^{+1400}_{-1360}	$0.49^{+0.19}_{-0.21}$	1041
$^{+0.6}_{-0.6}$	$3.5^{+0.6}_{-0.9}\times 10^{56}$	1030^{+320}_{-390}	$0.20^{+0.05}_{-0.07}$	308
$^{+0.4}_{-0.3}$	[illegible]	[illegible]	[illegible]	[illegible]
).04				



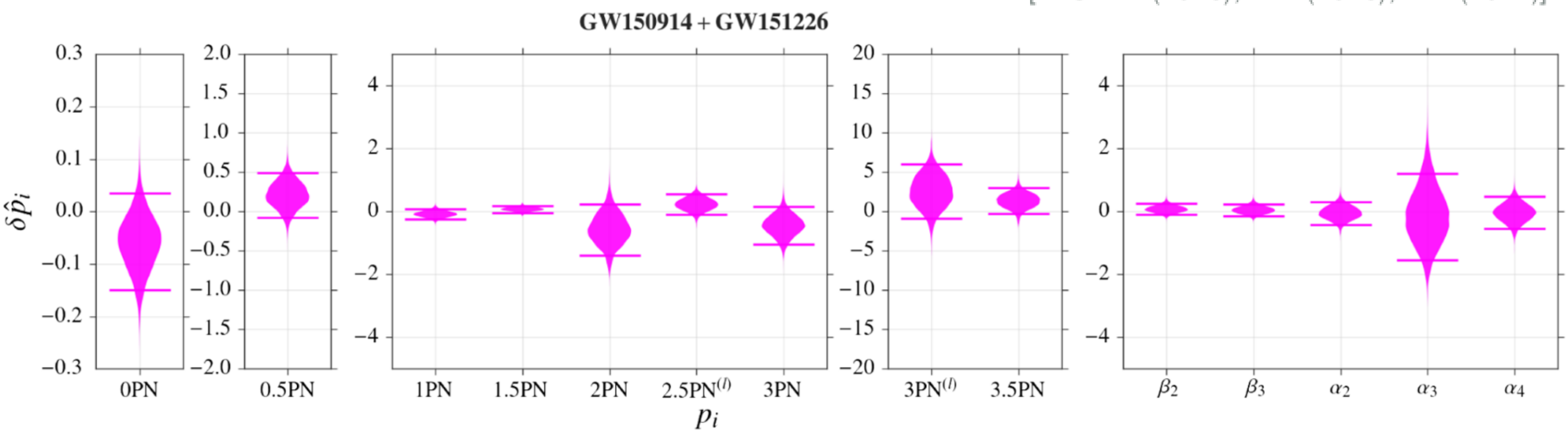
GW170818	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	$^{+0.5}_{-0.5}$
GW170823	$39.5^{+11.2}_{-6.7}$	$29.0^{+6.7}_{-7.8}$	$29.2^{+4.6}_{-3.6}$	$0.09^{+0.22}_{-0.26}$	$65.4^{+10.1}_{-7.4}$	$0.72^{+0.09}_{-0.12}$	$3.3^{+1.0}_{-0.9}$

Testing General Relativity

- Waveform: $h(f,\theta) = A(f;\theta)e^{i\phi(f;\theta)}$,
- $\phi = \phi_o + \sum \phi_k(\theta)(\pi M f)^{(k-5)}$
  $\theta = \{m_1, m_2, \mathbf{s_1}, \mathbf{s_2}\}$
- $\phi_k = \phi_k^{GR}(1+\delta\phi_k)$

[LVC PRL(2016), PRX(2016), PRL(2017)]

Post-Newtonian coefficients:



Conclusions

→ 2015: first detection of gravitational waves produced by a binary system of black holes
→ 2015-2017: additional detections (5 up to now) → initiate population studies
→ New class of stellar black holes (m > 15 M_{sun})
→ Parameter estimation