Lifetime and width of a particle

Unstable particles

A particle can decay into few (n) decay channels *i*

The transition rate for each *i* can be computed : Γ_i

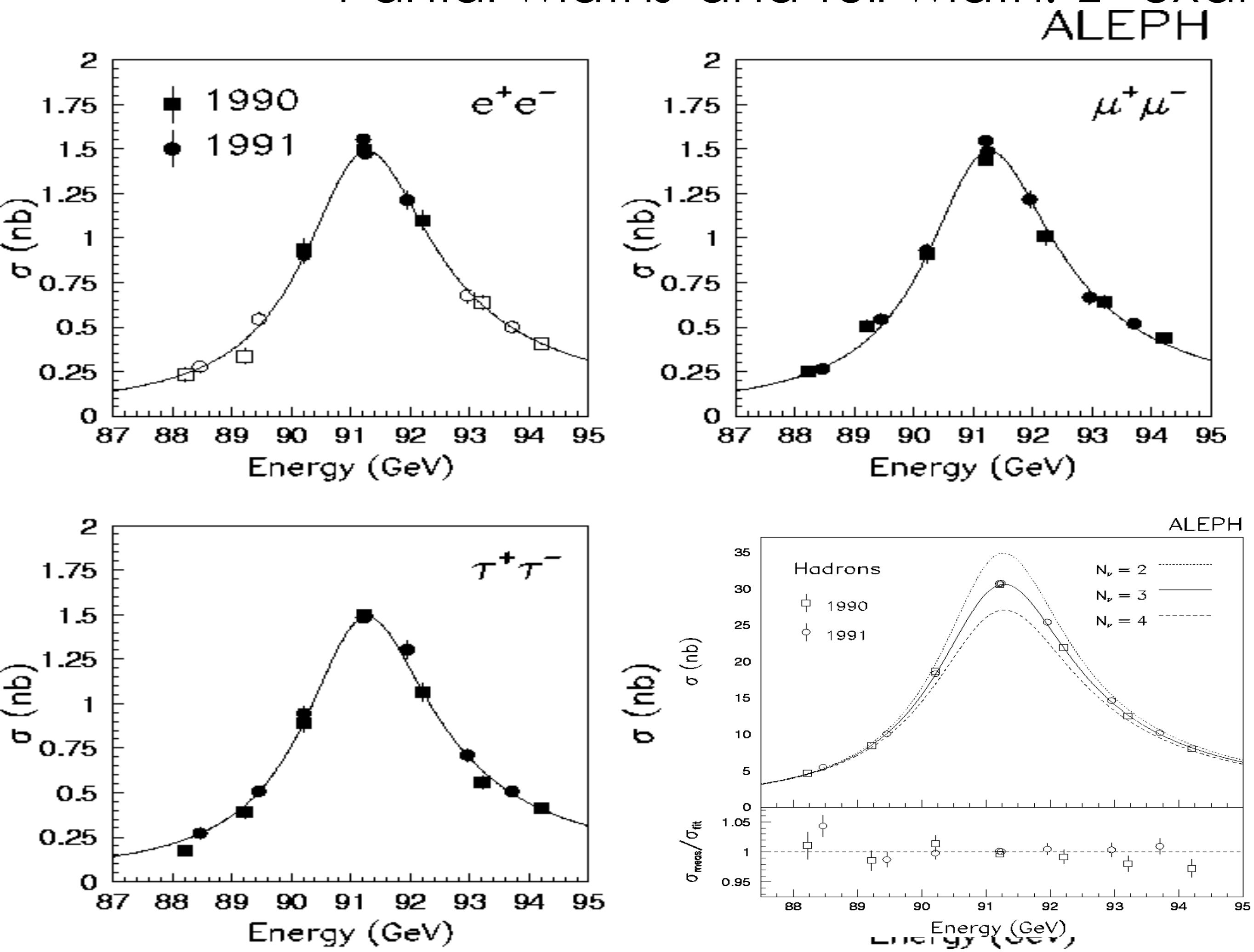
Starting with N particles, the number dN which decays during dt is

$$dN = -N \sum_{i=0}^{n} \Gamma_i dt = N\Gamma dt$$

$$\Rightarrow N(t) = N(0)e^{-\Gamma t} = N(0)e^{-t/\tau}$$

Definition :
branching ratios: $BR_i = \Gamma_i / \Gamma$ $\Sigma_i BR_i = 1$

Partial widths and full width: Z^0 example



Mode	BR_i
ee	~3.4%
μμ	~3.4%
ττ	~3.4%
hadrons	~70%

Lifetime and width

Unstable particle (finite lifetime) **+** Heisenberg : $\Delta E \cdot \Delta t \sim \hbar$

Δmc^2 → ΔE; τ → Δt

A set of identical unstable particles : measurement of their mass
⇒ range of values with a width Γ

$$\Rightarrow \quad \Gamma c^2 = \frac{\hbar}{\tau}$$ uncertainty in the rest energy = rate of its decay

The faster the decay, the larger the uncertainty of on *m*

Stable particle ↔ well defined mass state

Schrodinger Eq. (free particle with an energy E_0) : $i\hbar \frac{\partial\psi}{\partial t} = H\psi = E_0\psi$

$$\psi = ae^{-\frac{i}{\hbar}E_0 t} = ae^{-\frac{i}{\hbar}mc^2 t}$$

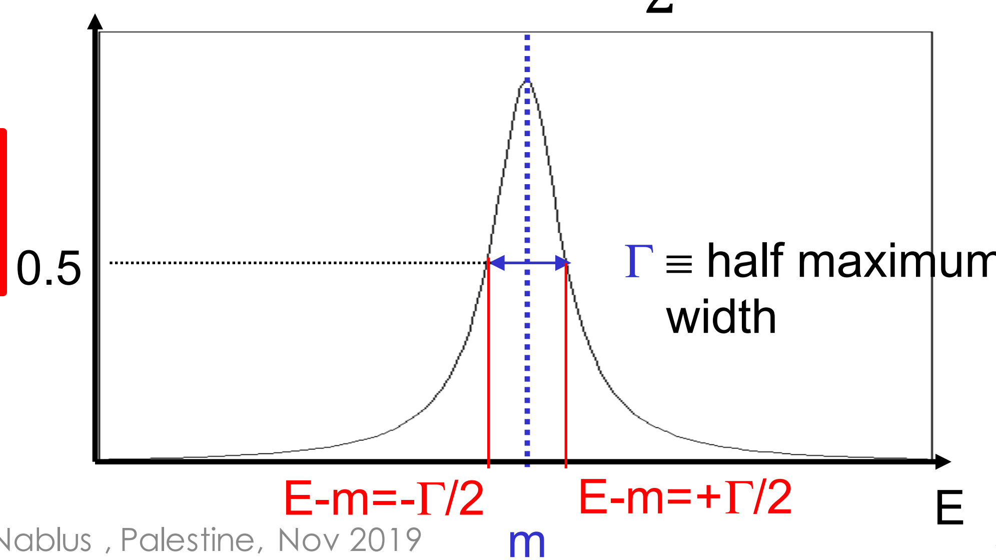
Stable particle : $|\psi(t)|^2 = |\psi(0)|^2 = |a_0|^2$

Unstable particle : $\psi(t) = a_0 e^{-\frac{ic^2}{\hbar}\left(m - i\frac{\Gamma}{2}\right)t} \Rightarrow |\psi(t)|^2 = a_0^2 e^{-\frac{t}{\tau}}$

Probability to find a state of energy E

$$A(E) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \psi(t) e^{\frac{-i}{\hbar}Et} dt \propto \frac{1}{(E - mc^2) + i\,\frac{\Gamma c^2}{2}}$$

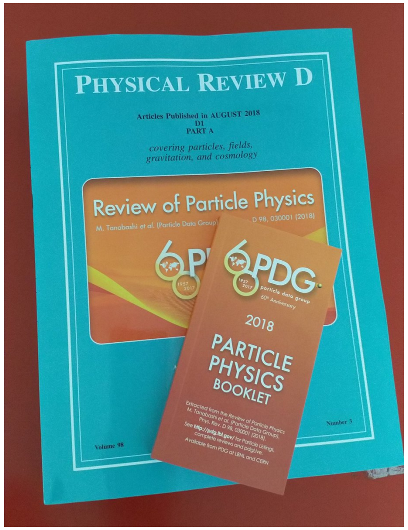
$$\Rightarrow |A|^2 \propto \frac{1}{(E - mc^2)^2 + \Gamma^2 c^4/4}$$



Numerically :

Lifetime	Width
10^{-23}s	65 MeV
10^{-17}s	6.5 eV
10^{-12}s	0.000065 eV

$$\hbar c = 197 \text{ MeV fm}$$



Particle	Mass	Width	Lifetime
K*	~892 MeV	~50 MeV	
π^0	~135 MeV		~ 8. 10^{-17} s
D_s	~1969 MeV		~0.5 10^{-12} s

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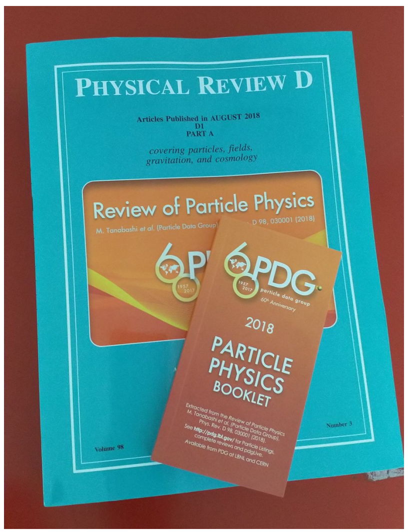


Particle	Mass	Width	Lifetime
K*	~892 MeV	~50 MeV	1.3 10^{-23} s
π^0	~135 MeV	8 eV	~ 8. 10^{-17} s
D_s	~1969 MeV	10^{-3} eV	~0.5 10^{-12} s

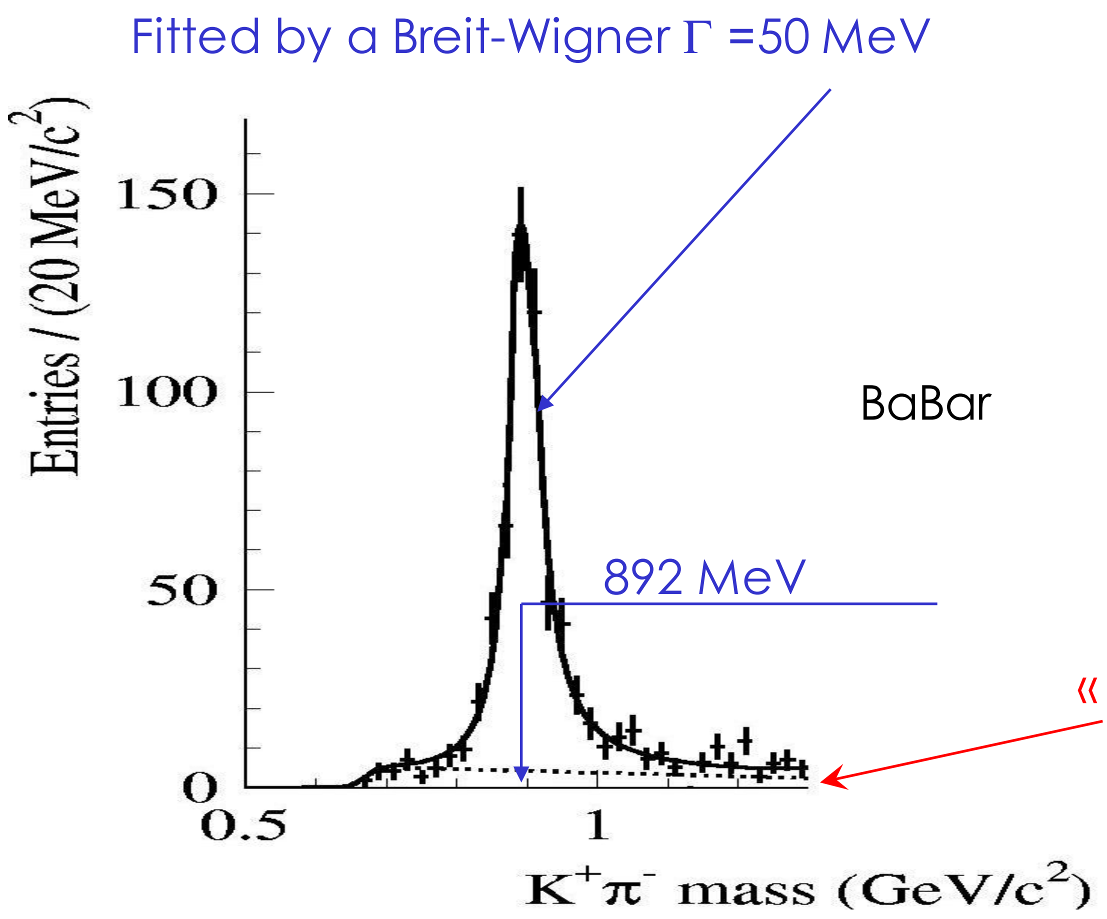
Computed from the measured values

Measuring widths, one is able to have information on very small lifetimes.

But how to measure the width and lifetimes ?

$K^-\pi^+$ experimental spectrum:

Search for a K^- and a π^+ in the detector and computation of the invariant mass



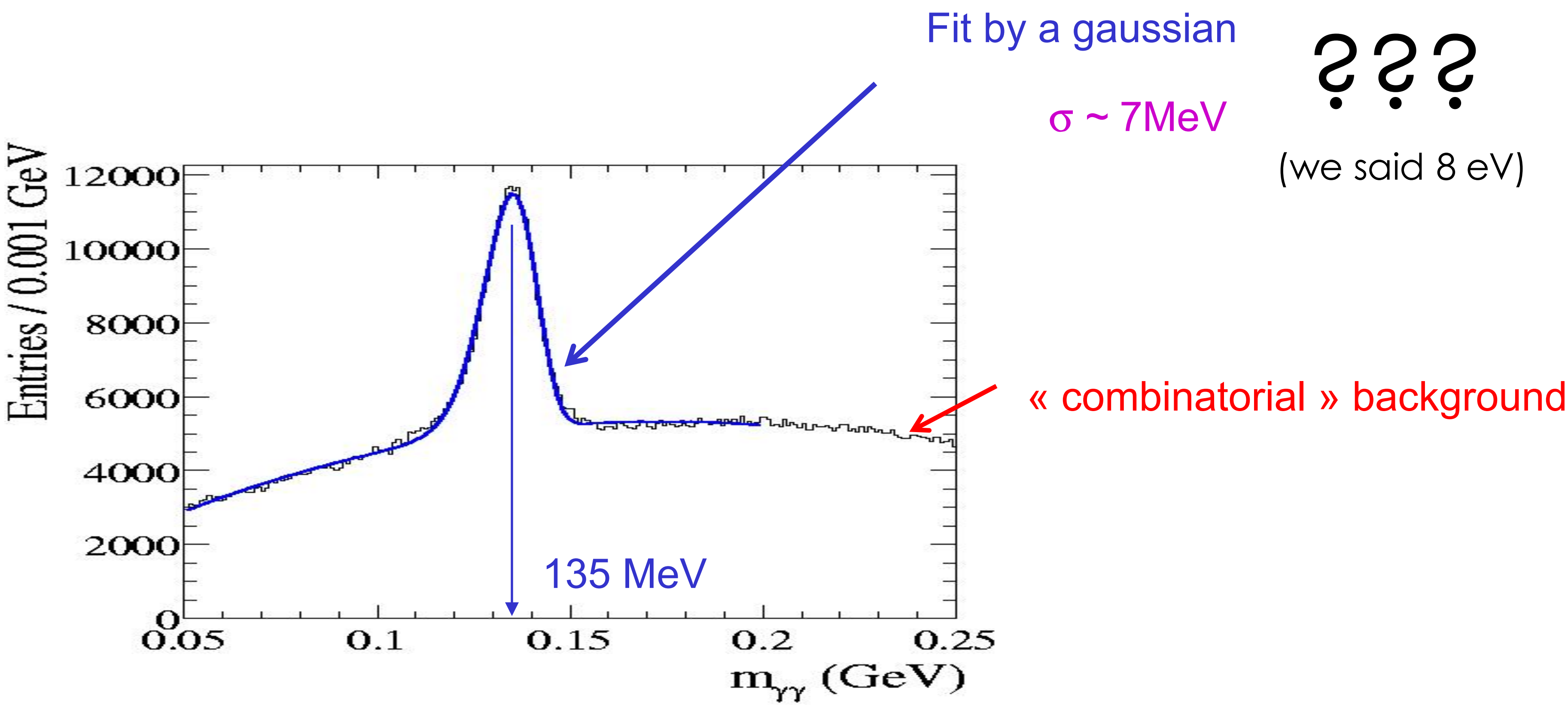
This is a K* !

« combinatorial » background

π^0 experimental spectrum :

2 γ reconstruction and computation of the invariant mass.

PDG → $\tau = 8.4 \times 10^{-17}$ s



???

(we said 8 eV)

Lifetime measurement

$\tau \sim 10^{-12}$ s

This is the particle lifetime in its rest frame

In the lab frame the particle is moving with v ~ c (β~1) ⇒ special relativity

$$L = \beta\gamma c\tau \sim \gamma c\tau$$

$$\gamma = \frac{E}{M}$$

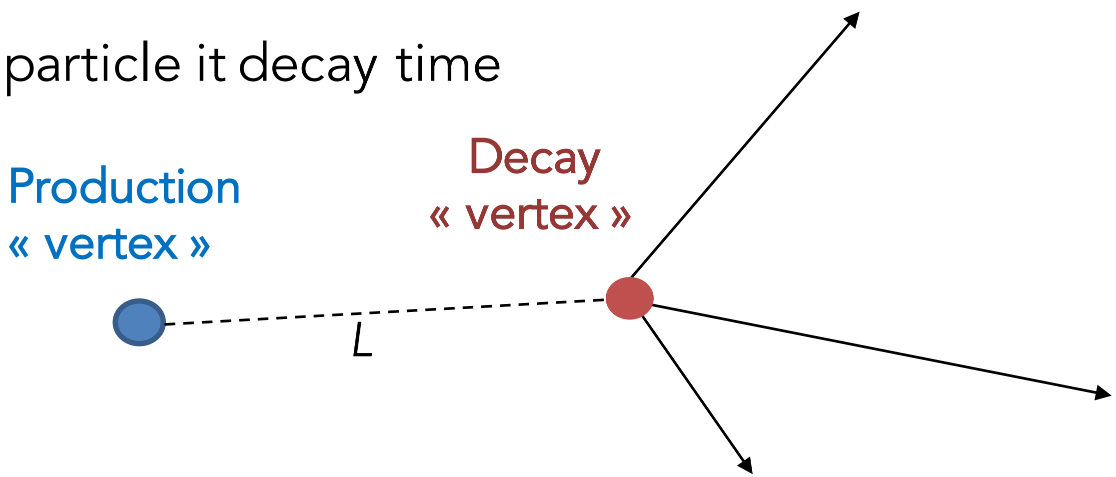
$$\beta^2 = 1 - \frac{1}{\gamma^2} = \left(\frac{v}{c}\right)^2$$

M = 5 GeV

	τ	$c\tau$	$\beta\gamma c\tau$
E = 5 GeV	10^{-12} s	0.3 mm	0.3 mm
E = 50 GeV	10^{-12} s	0.3 mm	3 mm

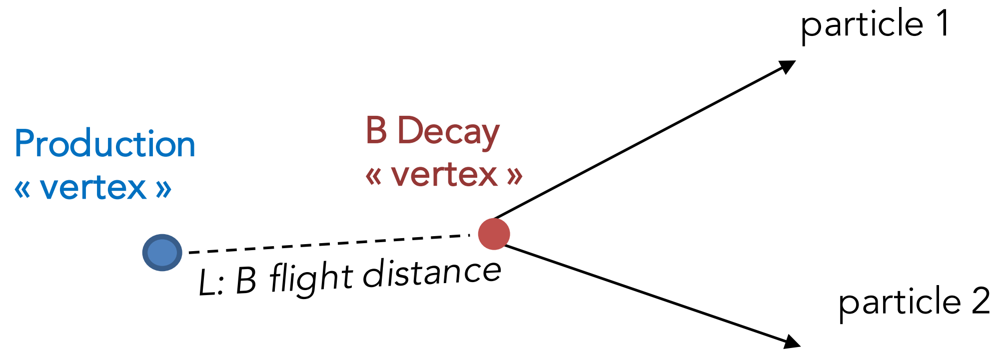
We want to measure for each particle it decay time

$$t = \frac{L}{v} = \frac{L \times M}{p}$$

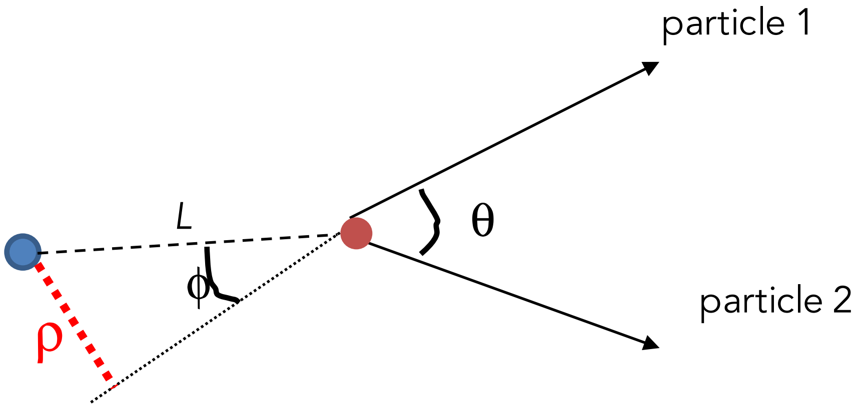


Lifetime measurements: impact parameter technique

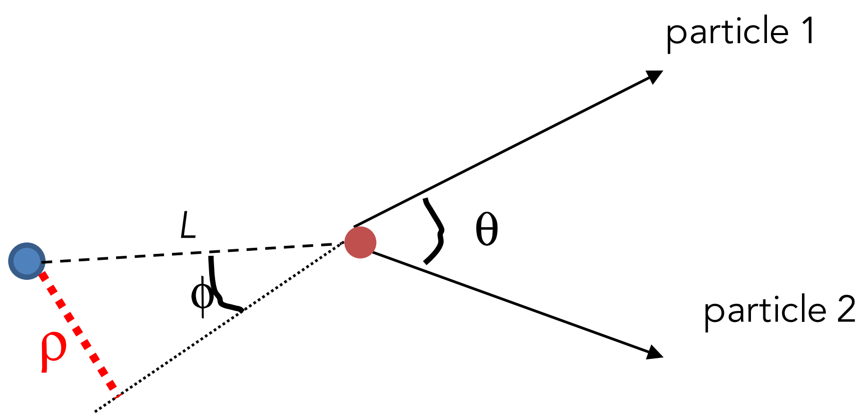
B→ 1 2



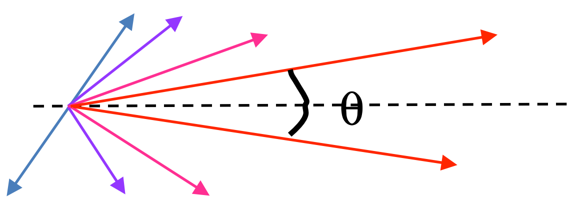
B→ 1 2



B→ 1 2



Large boost situation



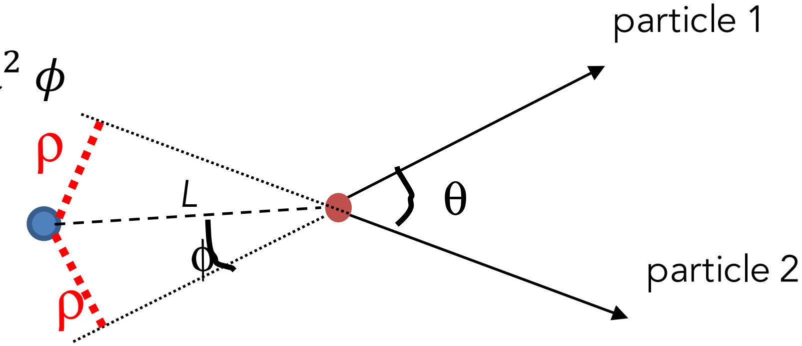
$$\frac{\theta}{2} \approx \phi$$

Energy – momentum conservation :

$$M_B^2 = M_1^2 + M_2^2 + 2E_1E_2 - 2p_1p_2\cos\theta$$

$E_i >> m_i$ i = 1,2

$$M_B^2 = 2E_1E_2(1-\cos\theta) = 4E_1E_2\,sin^2\frac{\theta}{2} = 4E_1E_2\,sin^2\,\phi$$



small angles ⇒ $\sin\varepsilon \sim \varepsilon$

$$M_B^2 = 4E_1E_2\phi^2 \approx E_B^2\phi^2$$

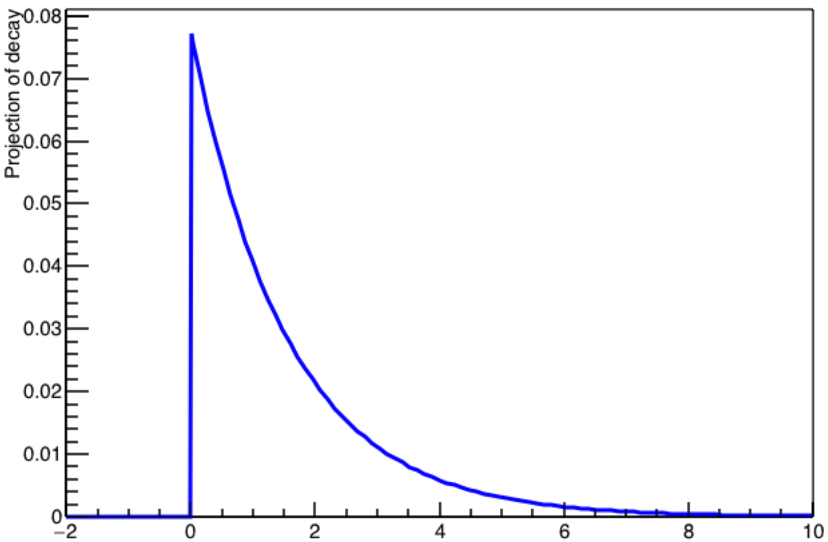
$$\phi = \frac{M_B}{E_B} = \frac{1}{\gamma}$$

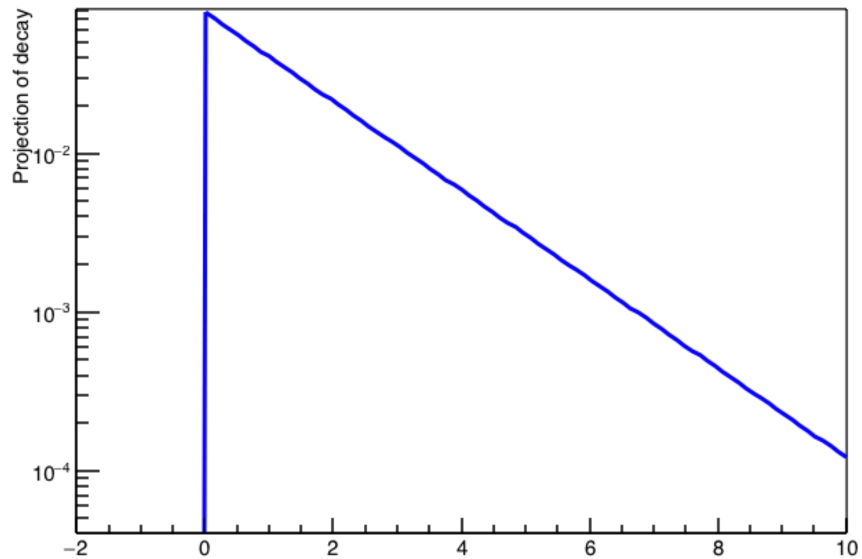
Special relativity : $L = \beta\gamma c\tau \sim \gamma c\tau$

$\rho = L\sin\phi \sim L\phi = \gamma c\tau\phi = c\tau$

The measurement of ρ = measurement of τ

True lifetime = 1.55 ps

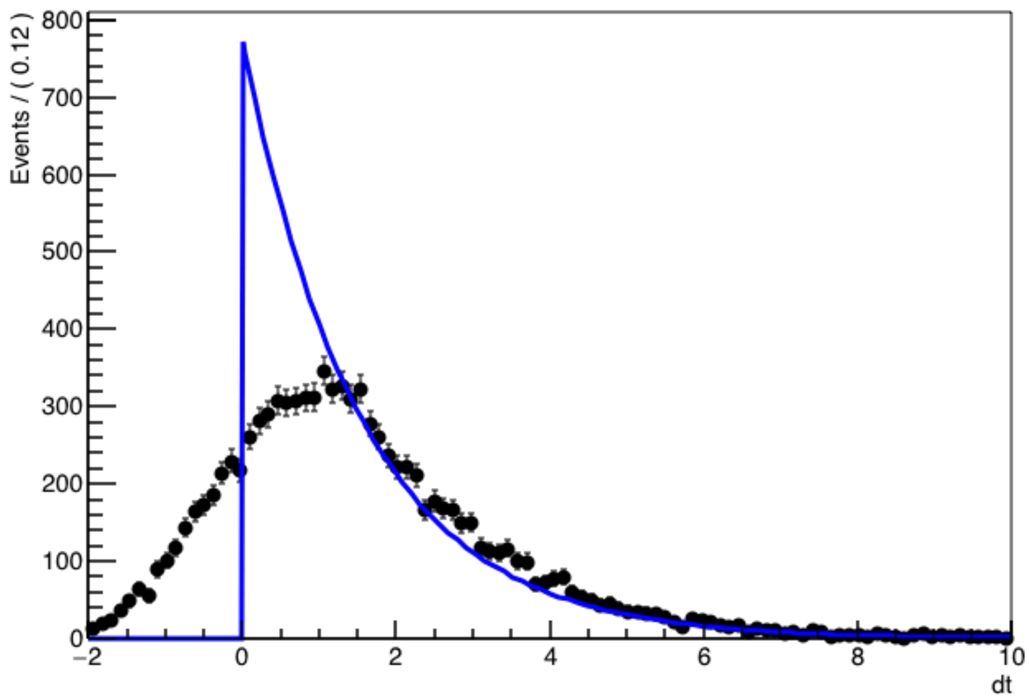




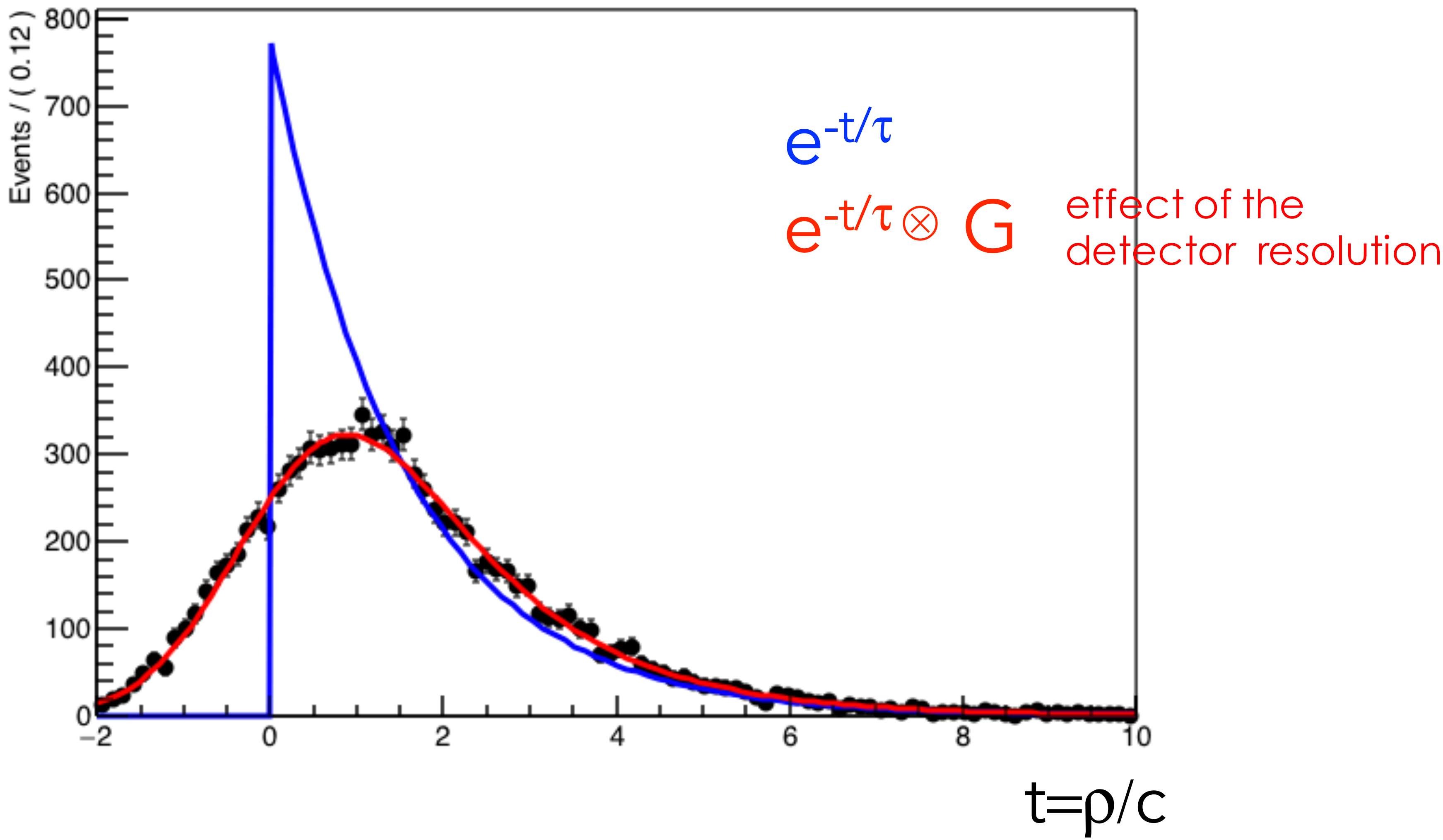
$\rho/c=t$

What we see !

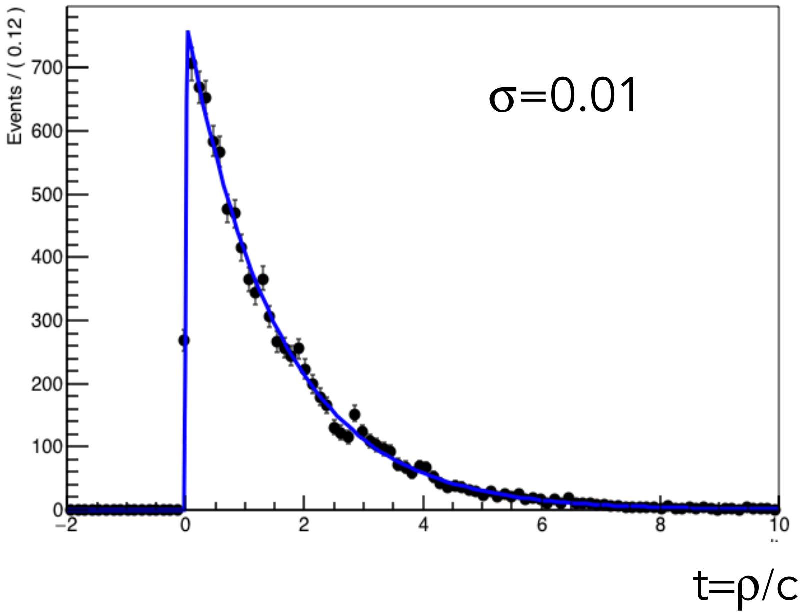
For each reconstructed decay :
- measure ρ
- compute t and fill the histogram

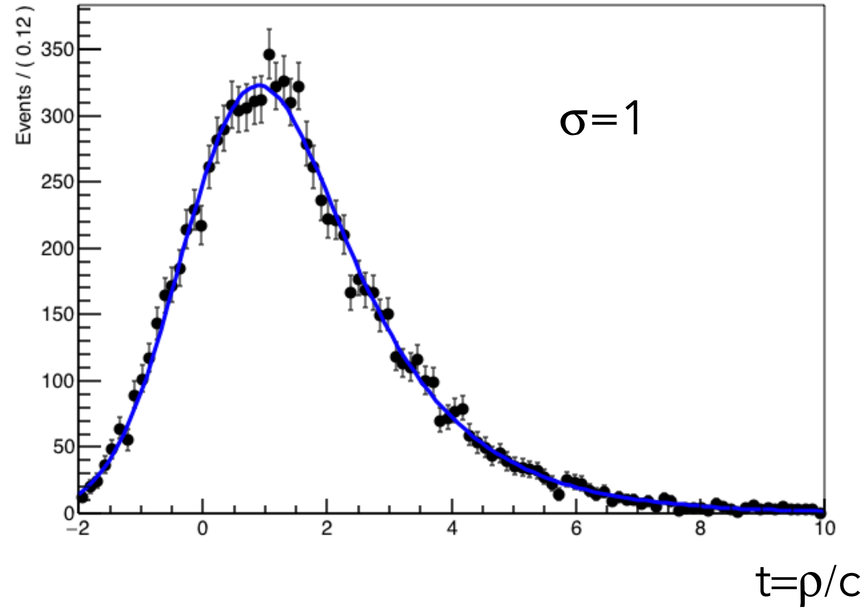


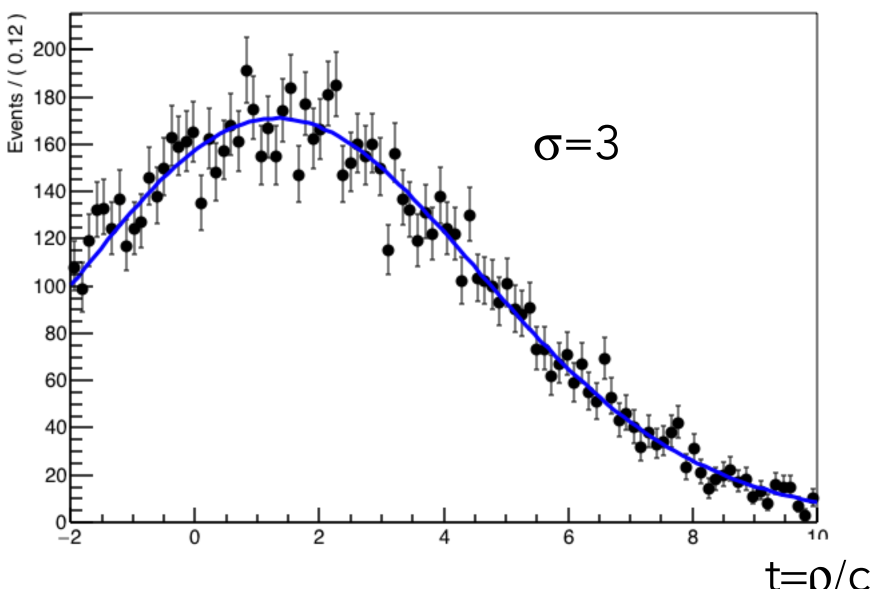
$\rho/c=t$



Events / (0.12)
800
700
600
500
400
300
200
100
0
−2
0
2
4
6
8
10
$e^{-t/\tau}$
$e^{-t/\tau} \otimes G$
effect of the
detector resolution
$t=\rho/c$



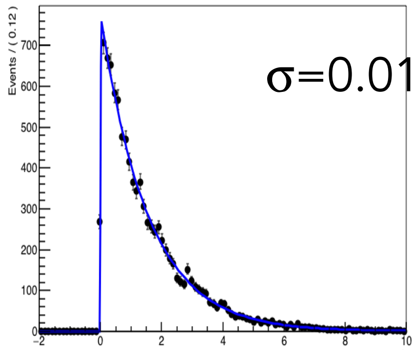




$$e^{-t/\tau} \otimes G$$

$$G \propto e^{\frac{(t-bias)^2}{2\sigma^2}}$$

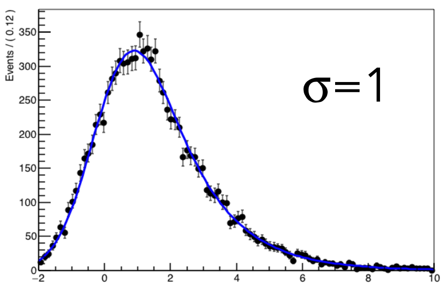
10000 events



tau = 1.561 +/- 0.016

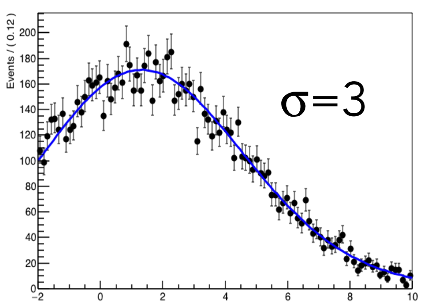
$t=\rho/c$

10000 events



tau = 1.552 +/- 0.019

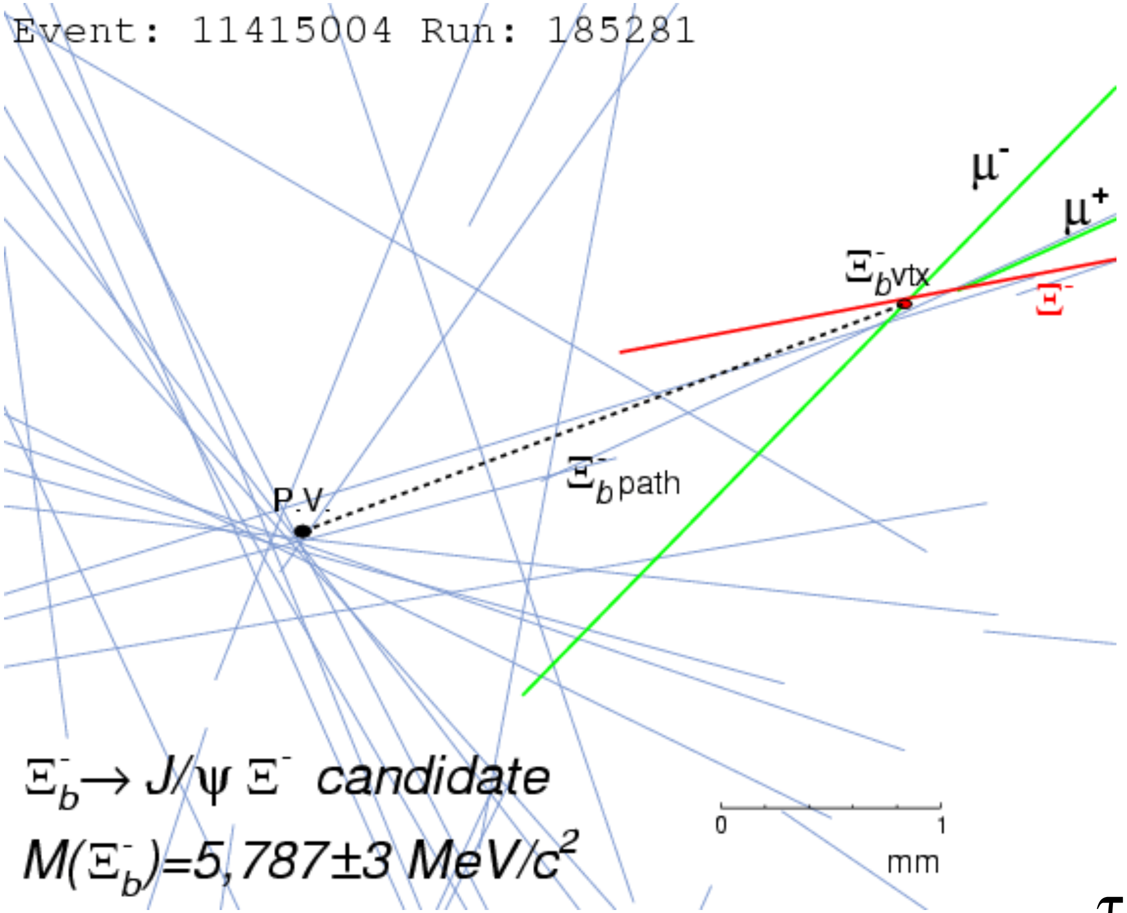
$t=\rho/c$



tau = 1.541 +/- 0.038

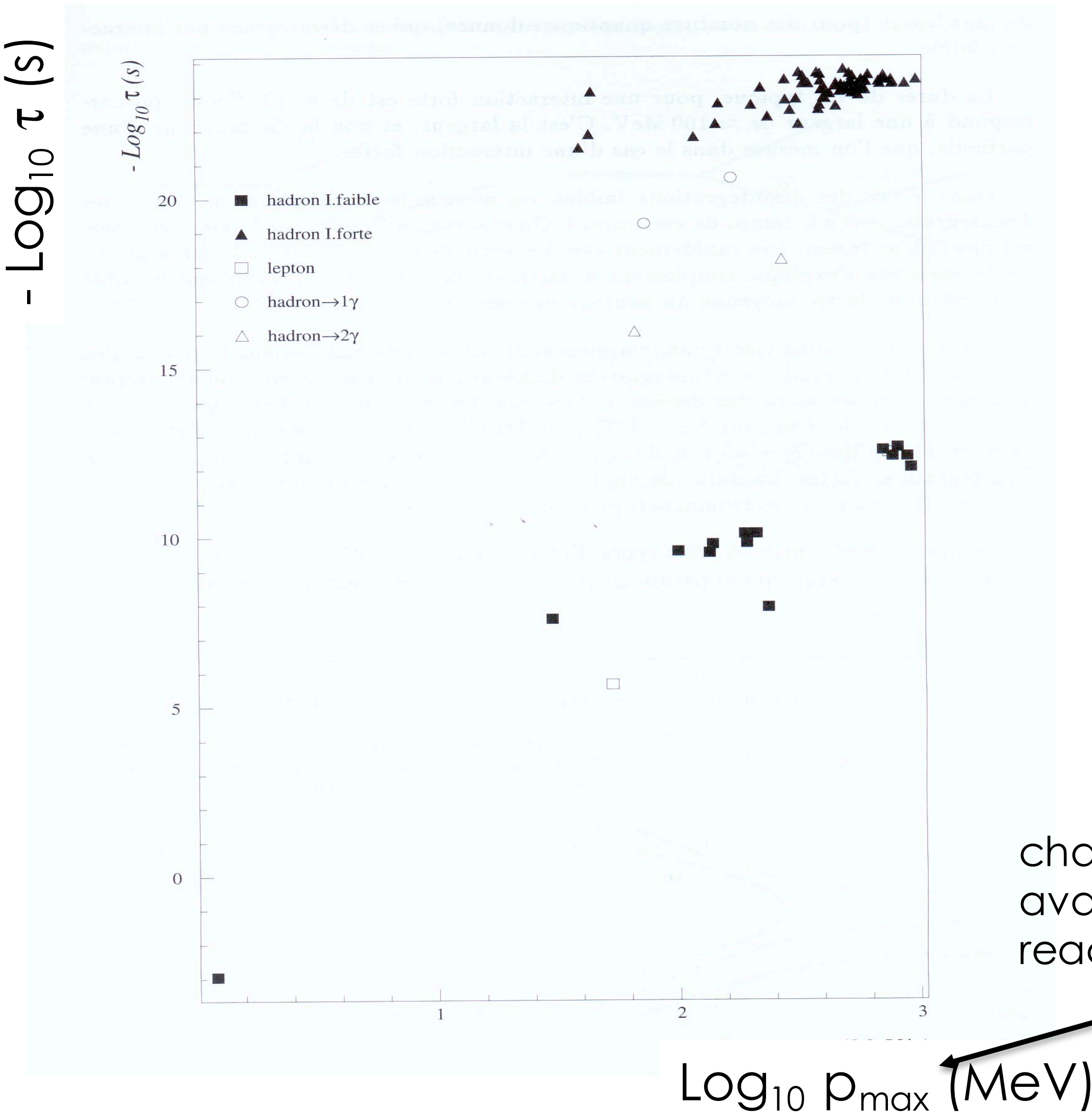
10000 events

$t=\rho/c$



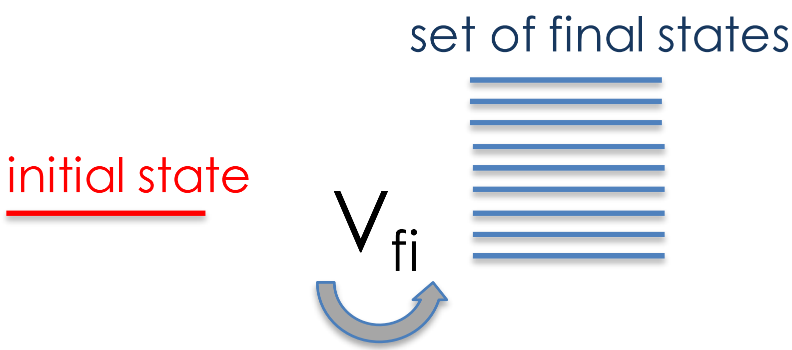
$\tau = (1.572 \pm 0.040) \times 10^{-12}$ s

Either from direct lifetime measurements or from translation of width measurements we observe a hugh variety of lifetimes !



characterize the available energy in the reaction

Fermi Golden rule



$$\Gamma = \frac{2\pi}{\hbar} |V_{fi}|^2 \frac{dN}{dE_f},$$

density of final states

Example of a spin 0 particle A decaying in B + C

$$dN = \frac{d^3\mathbf{p}_B}{(2\pi)^3}$$

When $\mathbf{p}_B$ is fixed due to (E,**p**) conservation (m_A known) $\mathbf{p}_C$ is fixed

No preferred direction $\Rightarrow$ $\Gamma = 2\pi\, |V_{fi}|^2 \frac{4\pi\, p_B^2}{(2\pi)^3} \frac{dp_B}{dE_f}$

$$|\mathbf{p}_B| = E_f/2 \quad \Rightarrow \quad \frac{dp_B}{dE_f} = \frac{1}{2} \quad \Rightarrow$$

$$\begin{aligned} \Gamma &= \frac{1}{2\pi} |V_{fi}|^2\, p_B^2 \\ &= \frac{1}{8\pi} |V_{fi}|^2\, m_A^2 \end{aligned}$$

- $\Delta^{++} \rightarrow p\,\pi$ ~ 10^{-23} sec
- $\Sigma^0 \rightarrow \Lambda\gamma$ ~610^{-20} sec
- $\pi^0 \rightarrow \gamma\gamma$ ~10^{-16} sec
- $\Sigma \rightarrow n\pi$ ~10^{-10} sec
- $\pi \rightarrow \mu\nu$ ~10^{-8} sec
- $n \rightarrow p\nu e$ ~15 minutes

strong
Electromagnetic

weak

$$\Gamma = \frac{1}{\tau} \propto \left|V_{fi}\right|^2 \propto \text{coupling constant}^{\,2}$$

Δ (uud) M~ 1230 MeV/c^2

Σ (uus) M~ 1190 MeV/c^2

Both Δ and Σ decay into nπ

Same final state and very similar phase space !

$$\frac{\tau(\Delta \to n\pi)}{\tau(\Sigma \to n\pi)} \approx 1\,?$$

Measurements: $$\frac{\tau(\Delta \to n\pi)}{\tau(\Sigma \to n\pi)} \approx \frac{10^{-23}s}{10^{-10}s}$$

$$\Gamma \propto \frac{1}{\tau} \propto |M|^2 \propto \sim |\text{coupling constant}|^2$$

$$\frac{\tau(\Delta \to n\pi)}{\tau(\Sigma \to n\pi)} = \frac{10^{-23}\text{ sec}}{10^{-10}\text{ sec}} = \left(\frac{\alpha_W}{\alpha_s}\right)^2$$

~same phase space

$$\frac{\alpha_W}{\alpha_s} \sim 10^{-6}$$

Lifetime as function of phase space