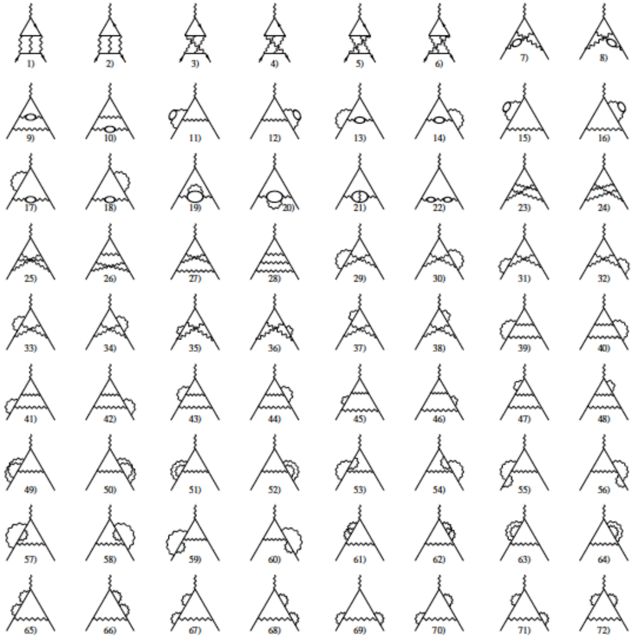
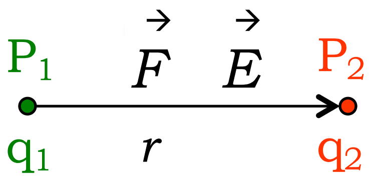
Introduction to Interactions
The electromagnetic interaction



Classical physics :

The particle P_1 creates around it a force field. If one introduces the particle P_2 in this field it undergoes the force.

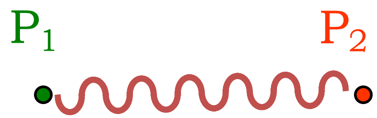
Electrostatic example :



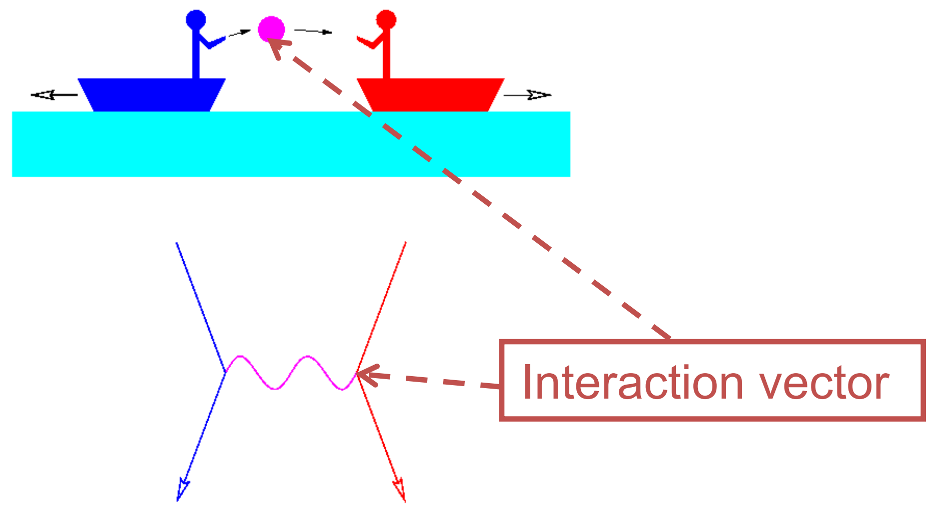
$$\vec{F} = q_2 \vec{E}(r) = q_2 \frac{kq_1}{r^2} \vec{u}_r$$

«modern» physics:

P_1 and P_2 exchange a field quantum; the interaction boson



The **heavier** the ball, the more difficult it will be to throw it **far away**



Range of the interaction $\propto$ 1/mass of the vector

Interaction in Quantum mechanics

- Creation and exchange of an interaction particle
  $\Rightarrow$ violation of the energy conservation principle during a limited time

$$\Delta t \approx \frac{\hbar}{\Delta E} = \frac{\hbar}{mc^2}$$

Heisenberg principle

- During Δt the particle can travel $R = c\,\Delta t$

$$R = \frac{\hbar c}{mc^2}$$

Range → « reduced » wave length (Compton)

with $\hbar c \cong 197.3\ MeV\ fm$

Example : an interaction particle with $m = 200\ MeV \Leftrightarrow R = 1\ fm$

More details and somehow more formally : Shape of the interaction potential

Klein-Gordon equation for a spin 0 particle :

$$E^2 = p^2c^2 + m^2c^4$$

operators

$$E = i\hbar\frac{\partial}{\partial t}$$

$$p = -i\hbar\nabla$$

$$(i\hbar)^2\frac{\partial^2\psi}{\partial t^2} = (i\hbar)^2 c^2\nabla^2\psi + m^2c^4\psi$$

$$-\frac{\partial^2\psi}{\partial t^2} = -c^2\nabla^2\psi + \frac{m^2c^4}{\hbar^2}\psi$$

$$\Rightarrow \nabla^2\psi - \frac{m^2c^2}{\hbar^2}\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0$$

(one only deals with stationary states)

$$\nabla^2\psi - \frac{m^2c^2}{\hbar^2}\psi = 0$$

In spherical symmetry : ψ=U(r) and ΔU(r)=∇²U(r)= $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU(r)}{dr}\right) = \frac{m^2c^2}{\hbar^2}U(r)$

if $m \neq 0$:

$$U(r) = -\frac{g^2}{r}e^{-r/R}$$

$r>0$ Yukawa potential

g coupling constant

$$R = \frac{\hbar}{mc}$$

Range

if $m=0$:

$$\Delta U(r) = 0$$

$$U(r) = -\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

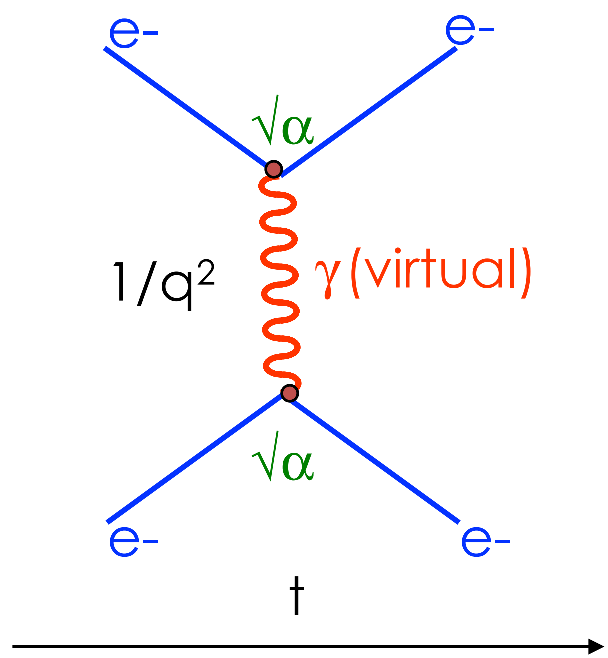
$r>0$

q_i = charge

In this case the Yukawa potential is equivalent to the Coulomb one

Interactions in Quantum Field theory

- Between charged particles
- One charge : the electric charge
- Vector of the interaction : the photon (γ)
- An example of a Feynman graph for QED:



electrons exchanging a photon

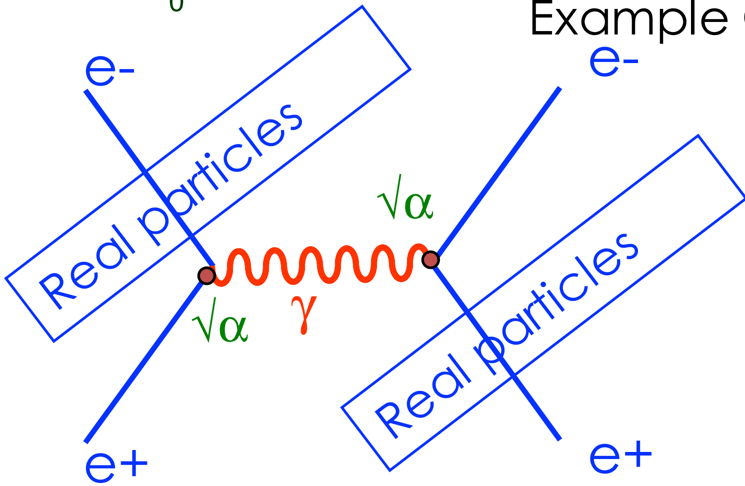
or

An e^- which emits a γ and moves back. The γ is absorbed by an other e^- whose direction is modified

Virtual particles

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 hc} = \frac{1}{137}$$

Example QED : e^+e^- symmetric collision in the rest frame



$$E_{e+} + E_{e-} = E_\gamma$$

$$\vec{p}_+ + \vec{p}_- = \vec{p}_\gamma$$

$$m_\gamma^2 = 2m_e^2 + 2E_{e+}E_{e-} - 2p_+p_-\cos\theta$$

In the rest frame : $\vec{p}_+ + \vec{p}_- = \vec{p}_\gamma = \vec{0}$

$$\theta = \pi \Rightarrow m_\gamma^2 = 2m_e^2 + 2E_{e+}E_{e-} + 2p_+p_-$$

incompatible with $m_\gamma = 0$

The γ is « off-shell »

It can be interpreted as :

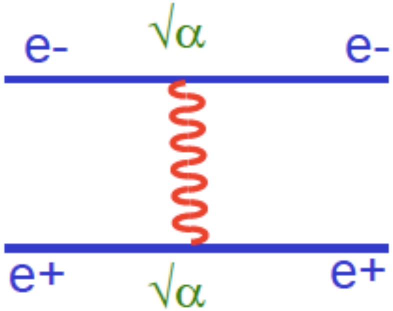
Violation of the energy-momentum conservation law

Or

Creation of a massive virtual photon during a « short » time

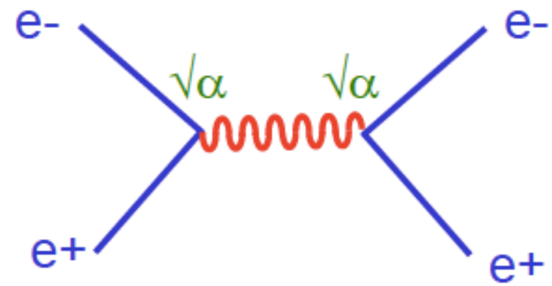
the γ can only exist virtually thanks to $\Delta E.\Delta t \approx \hbar$

$e^+e^- \rightarrow e^+e^-$ interaction



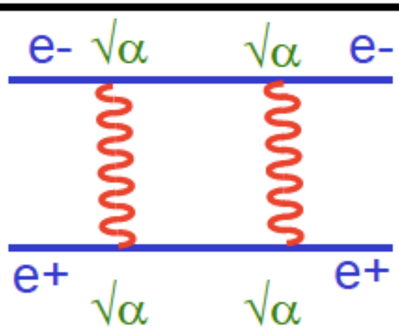
γ exchange between an e+ and an e-

+

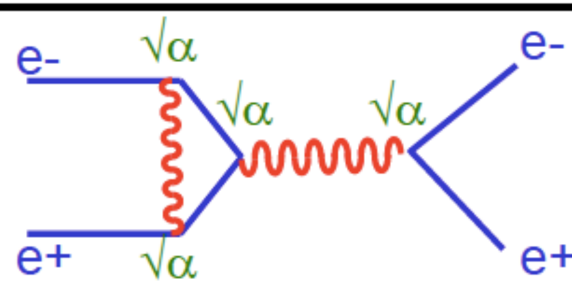


e+e- pair annihilation in γ and γ conversion in an e+ and an e-

$\sim\alpha$



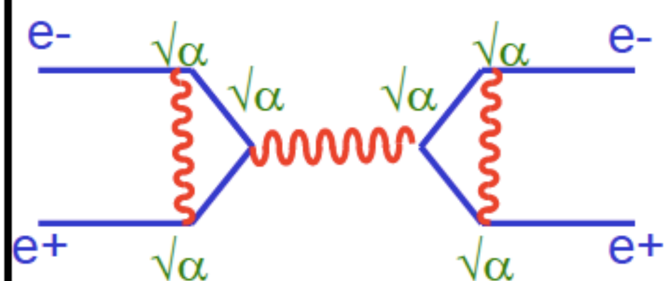
+



+ …

$\sim\alpha^2$

exchange of 2 γ between an e+ and an e-



+ …

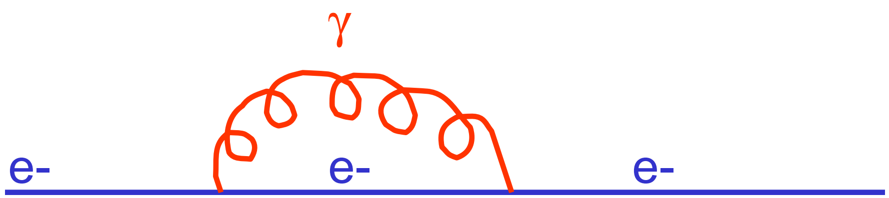
$\sim\alpha^3$

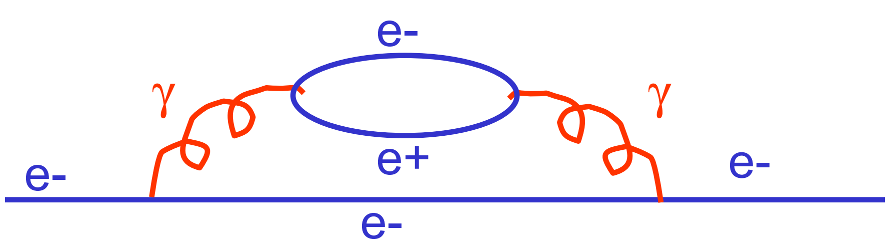
α small (1/137) : one can develop in perturbation series

The way we see the electron and the photon is modified

electron :

The electron emits and absorbs all the time virtual γ, it can be seen as :

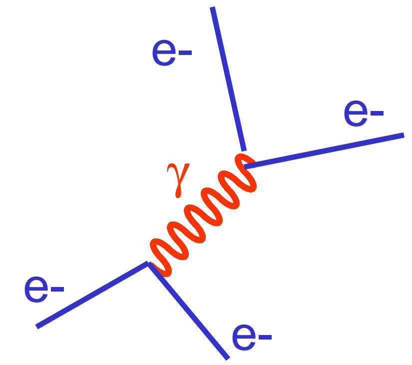


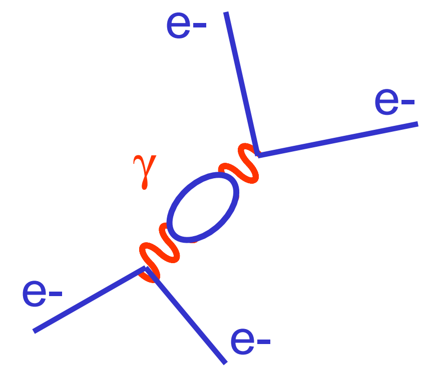


…

=> Theoretical (α « running ») and experimental (g-2) consequences

photon :





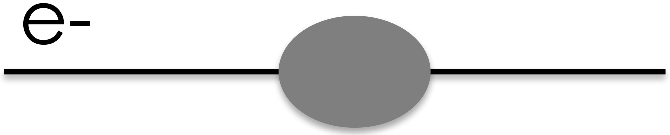
Anomalous magnetic moment of the electron

e-

Dirac eq $\vec{\mu} = g(e/2m)\vec{s}.$

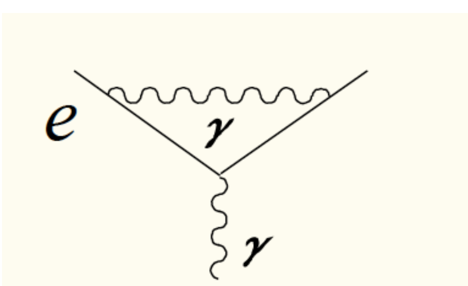
$$a = \frac{g-2}{2} = 0$$

e-



$a \neq 0$

First order (1948) :

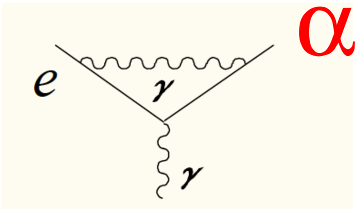


$$a = \frac{g-2}{2} = \frac{\alpha}{2\pi}$$

But in fact …

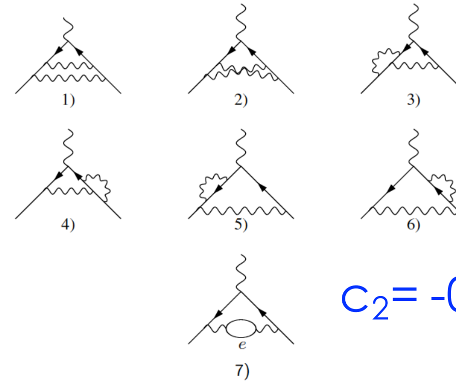
$$a_e^{SM} = \sum_{n=1}^{5} c_n \left(\frac{\alpha}{\pi}\right)^n$$

α



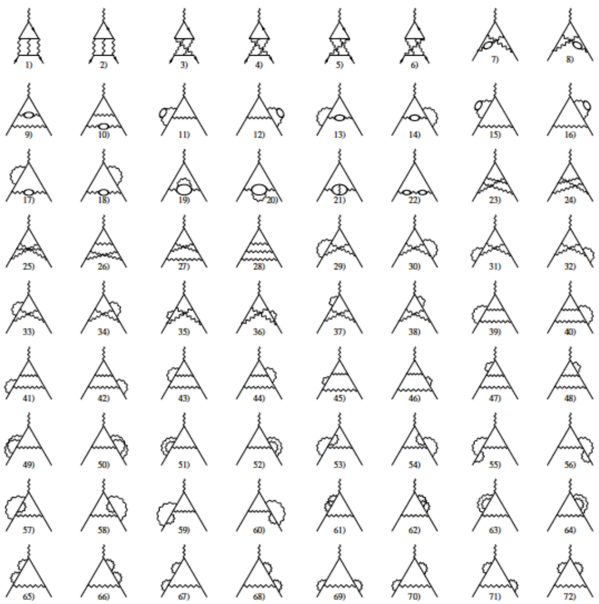
$c_1 = 1/2$

α^2



$c_2 = -0.3284789\ldots$

α^3



$c_3 = 1.181241\ldots$

α^4 891 diagrams

$c_4 = -1.9106$

α^5 12672 diagrams

$c_5 = 9.16$

numerical calculations

$\alpha = 1/137$

$$a_e^{\mathrm{SM}} = \sum_{n=1}^{5} c_n \left(\frac{\alpha}{\pi}\right)^n$$

dominant

$$+2.7478(2) \times 10^{-12} \quad \text{[Loops in QED with } \mu, \tau\text{]}$$

$$+0.0297(5) \times 10^{-12} \quad \text{[weak interactions]}$$

$$+1.682(20) \times 10^{-12} \quad \text{[strong interactions / hadrons]}$$

$$a^e_{\mathrm{exp}} = 1159652180.91(26) \times 10^{-12}$$

$$a^e_{\mathrm{theory}} = 1159652181.78(77) \times 10^{-12}$$

Amazing agreement : **experimental proof of QED, namely the fact that the electron is surrounded od its interactions**